# RESONANCE CHARACTERISTIC OF THE HYDRAULIC SYSTEM OF A WATER POWER PLANT

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#### 1. Introduction

The hydraulic system of a water power plant which is composed of a reservoir, a tunnel and a surge tank, and forms the so-called U-tube system as shown in Fig. 1 has its own resonance characteristic. If the change of the water flow to the turbine acts on this system as a disturbance, the water level of the surge tank oscillates up and down according to its intrinsic resonance characteristic. As the amount of water flow supplied to the turbine must be kept so that the tank water level lies within a limited range, it is needed to get the frequency characteristic of the hydraulic system in order to set the operating standard of the power plant. Especially for the power plants used for the load regulation, it is necessary. The frequency characteristic of the hydraulic system may also be useful for estimating the increase of a loss factor in the tunnel.

In this paper, we shall treat the estimating of the frequency response function of the hydraulic system based upon the observed data.

#### 2. Types of surge tank

Types of the surge tank used at present are as follows:
simple type,
differential type,
orifice type,
chamber type,
combination of the above-mentioned types.

Among these types the simple type is convenient for theoretical analysis, because of its simple dynamic properties. As for the hydraulic systems with other types of tank, similar analysis can be made by introducing the concept of an equivalent loss factor, because the shapes of their resonance characteristic curves are identical with those of the systems with a simple type tank.

# 3. Equation of the hydraulic system with a simple type surge tank

The equation expressing the dynamic state of the hydraulic system

are given as follows (see Fig. 1). Equation of continuity:

$$(1) Q = Q_t + Q_c$$

where Q,  $Q_t$ ,  $Q_c$  denote amounts of water flow at the places shown in Fig. 1 (m<sup>3</sup>/s).

Equation of motion:

$$\frac{L}{g \cdot f} \frac{dQ}{dt} \pm \frac{\alpha}{f^2} Q^2 = -H_t \qquad \left\{ \begin{array}{l} Q > 0 + \\ Q < 0 - \end{array} \right.$$

where f: sectional area of the tunnel ( $m^2$ ),

L: length of the tunnel (m),

g: acceleration of gravity (9.8 m/s<sup>2</sup>),

 $H_t$ : difference of water levels between the reservoir and tank (m),

 $\alpha$ : a term expressing the attenuation of the hydraulic system (m-s<sup>2</sup>).

Equation of the tank water level:

$$F_t \frac{dH_t}{dt} = Q_t$$

where  $F_t$  denotes the sectional area of the tank.

Let  $\Delta Q$  denote the deviation of the amount of water flow from the initial value i.e.  $Q=Q_i+\Delta Q$  where  $Q_i$  is the initial value. When this  $\Delta Q$  is comparatively small, we can consider

$$Q^2 = Q_i^2 + 2Q \Delta Q$$
.

When the amount of deviation from the initial state is observed, the preceding equations are written as follows, small letters standing for the deviation:

$$\left\{egin{array}{l} q=q_{\iota}+q_{c} \ T_{v}rac{dq}{dt}+K_{v}q=-h_{\iota} \ T_{\iota}rac{dh_{\iota}}{dt}=q_{\iota} \end{array}
ight.$$

where

$$T_v = \frac{L}{gf}$$
  $K_v = \frac{2\alpha Q_i}{f}$   $T_t = F_t$ .

From equations (4), the transfer function between  $h_t$  and  $q_c$  is derived,

(5) 
$$Y(s) = \frac{h_t}{q_c} = -\frac{T_v s + K_v}{T_t T_v s^2 + K_v T_t s + 1}$$

where s is a differential operator.

# 4. Estimation of the frequency response function by means of a statistical method

# 4.1 Parameters of the hydraulic system

maximum power of the plant: 90 MW sectional area of the tunnel,  $f: 19.635 \text{ m}^2$  length of the tunnel, L: 3,208 m sectional area of the tank,  $F_t: 121.3 \text{ m}^2$ 

(For the differential type tank, the sum of the sectional areas of the tank and riser is used.)

#### 4.2 Measurement

The output power of the plant and the water level of the tank are recorded by an oscillograph for about four hours. From these records necessary data were sampled. In this analysis the output-power of the power plant is measured as the system input instead of the turbine water flow, because the continuous measurement of the hydraulic flow is technically difficult, and the hydraulic flow in the turbine seems to be approximately proportional to the electric power of the plant in the observed range.

#### 4.3 Treatment of data

sampling interval,  $\Delta t = 13.14 \text{ sec}$  number of the sampled data, M = 1,000 maximum lag in the calculation of the correlogram, H = 199 number of divisions of the power spectra, h = 68

### 4.4 Estimated frequency response function

Figures 2 through 6 show the estimated correlograms, power spectra and frequency response function together with coherency and confidence limits.

# 5. Comparison of the estimated frequency response function with the theoretically calculated results

In equation (5),  $K_v$  is the only unknown parameter. However, this

may be evaluated from the statistically estimated amplitude gain and phase shift so that the curves calculated by equation (5) coincide with the estimated ones at their crest value of the amplitude and in their phase shift. In Fig. 6 the characteristic curves with  $K_v$  thus obtained are given in a solid line. As shown in this figure, the estimated points lie around the theoretical curves both in the amplitude gain and in the phase shift in the necessary frequency range, and the favorable value of coherency and the narrow bandwidth of confidence limits therein suggest that the present estimate of the frequency characteristic of the hydraulic system is fairly satisfactory.

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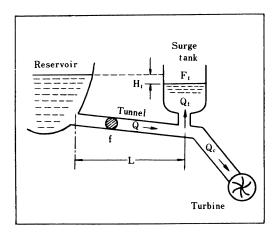


Fig. 1 Hydraulic system of a water power plant

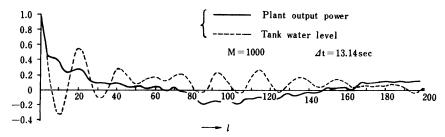


Fig. 2 Auto-correlogram of the plant output power and tank water level

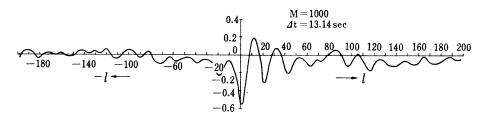


Fig. 3 Cross-correlogram

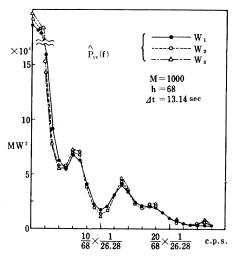


Fig. 4(a) Estimated power spectrum of the plant output power

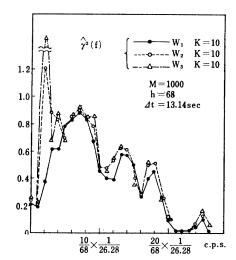


Fig. 5 Coherency

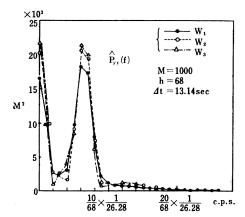


Fig. 4(b) Estimated power spectrum of the tank water level

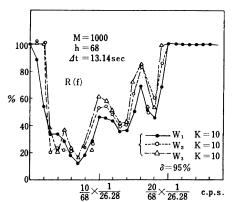


Fig. 6 Approximate confidence limits for  $\delta$ =95%

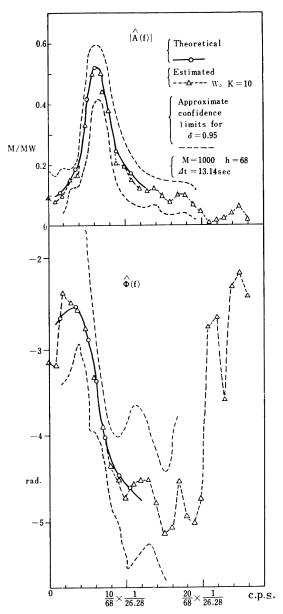


Fig. 7 Estimates of the amplitude gain and phase shift with 95% confidence limits