5

FREQUENCY RESPONSE OF AN AUTOMOBILE ENGINE MOUNTING

BY ICHIROH KANESHIGE

1. Introduction

As is well known, an automobile engine today is in most cases a reciprocating machine, and this causes vibration. To prevent the vibration being transmitted from the engine to the body we usually use rubber mountings. In this case, the transmission of the vibration, of course, depends on the rubber mounting. Therefore, we have to select a suitable engine mounting system considering the frequency of the exciting force, that is, the frequency of the engine explosion.

Now, the equation of motion of the engine mounting system is given by equation (1), and the eigenvalues of the system are given as the roots of equation (2).

$$(1)\qquad \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & \cdot & \cdot & K_{26} \\ \cdot & \cdot & K_{33} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & K_{44} & \cdot & \cdot \\ K_{51} & \cdot & \cdot & \cdot & K_{55} & \cdot \\ K_{61} & \cdot & \cdot & \cdot & \cdot & K_{66} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} F_x - M\ddot{x} \\ F_y - M\ddot{y} \\ F_z - M\ddot{z} \\ N_x - I_x\ddot{\phi} \\ N_y - I_y\ddot{\theta} \\ N_z - I_z\ddot{\psi} \end{bmatrix},$$

$$(2)\qquad \begin{vmatrix} K_{11}-M\omega^2 & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22}-M\omega^2 & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33}-M\omega^2 & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44}-I_x\omega^2 & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55}-I_y\omega^2 & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66}-I_z\omega^2 \end{vmatrix} = 0 ,$$

where (see Fig. 1)

x, y, z : deflections along the principle axes, X, Y, Z, of the moment of inertia of the system,
ϕ, θ, ψ : angles of rotation about the axes X, Y, Z,
K_{ij} : coefficients of the engine mounting stiffness,
F_x, F_y, F_z : external forces applied along the axes X, Y, Z,
N_x, N_y, N_z : external moments applied about the axes X, Y, Z,
M : mass of the engine assembly,
I_x, I_y, I_z : principal moments of inertia,
ω : circular frequency.

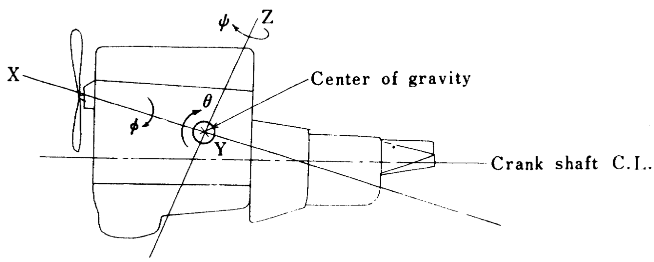

Fig. 1

Usually some vibration modes are coupled, namely some $K_{ij}\,(i\neq j)$ are not zero. In Table 1, examples of the engine mounting eigenvalues and eigenvectors are given.

Let $f(\omega)$ be the vibration transfer function of an engine mounting system, $P(\omega)$ the exciting force from the engine, and $\pi(\omega)$ the power spectrum of the excitation at the body side engine mounting bracket, caused by the road surface roughness. Then the vibration $B(\omega)$ transmitted from the engine to the body is

$$B(\omega)=f(\omega)\cdot P(\omega) \tag{3}$$

and the power spectrum $E(\omega)$ of the engine vibration caused by the road surface roughness is

$$E(\omega)=|f(\omega)|^2\cdot\pi(\omega)\,. \tag{4}$$

It is desirable from the point of view of ride comfortableness, that the value of $B(\omega)$ is small, and from the point of view of engine mounting strength, that the value of $E(\omega)$ is small. For this purpose we have to select a suitable $f(\omega)$, that is, a suitable engine mounting system. However, it is very hard to get a sufficiently precise estimate of $f(\omega)$ from only the design for a mounting system. To get such an estimate of $f(\omega)$ for a particular mounting system, it is necessary to measure $f(\omega)$

from an actual vehicle with that mounting system. By carrying out such a measurement on variant mounting systems, we can approach an optimal mounting system. Thus, to estimate $f(\omega)$ of a particular mounting system from actual data becomes our main concern. In fact, we estimate the transfer function from records of vibration of the body and engine, by the cross spectral method. To give a description about this is the purpose of this paper.

2. Procedure of the measurement

Even the engine with 6 degrees of freedom, by itself, is too complex to understand without a very extensive measurement and a lot of vibration pickups, so it is impractical to analyze this system as it is. But problems of the actual vehicle are fairly restricted. For example, in the problem of engine mounting strength we need only vertical vibrations in the 1~20 c.p.s. range. Therefore, it would be sufficient to measure $E_v(\omega)$ $(2\pi \leqq \omega \leqq 40\pi$ rad/sec), the power spectrum of an approximate system which is concerned only with vertical vibrations. By this reason we consider a system with 3 degrees of freedom as shown in Fig. 2, as a model for the original system.

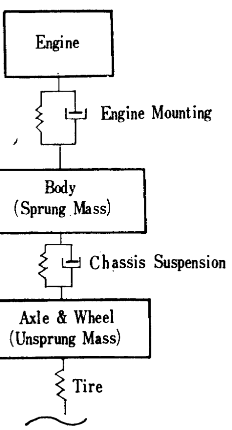

Fig. 2

M_E : mass of the engine assembly.
K_E : stiffness of the engine mounting.
C_E : damping coefficient of the engine mounting.
M : mass of the sprung mass.
K : stiffness of the chassis suspension.
C : damping coefficient of the chassis suspension.
M_T : mass of the unsprung mass.
K_T : stiffness of the tire.

In Fig. 3 block diagrams of measurement and analysis are shown.

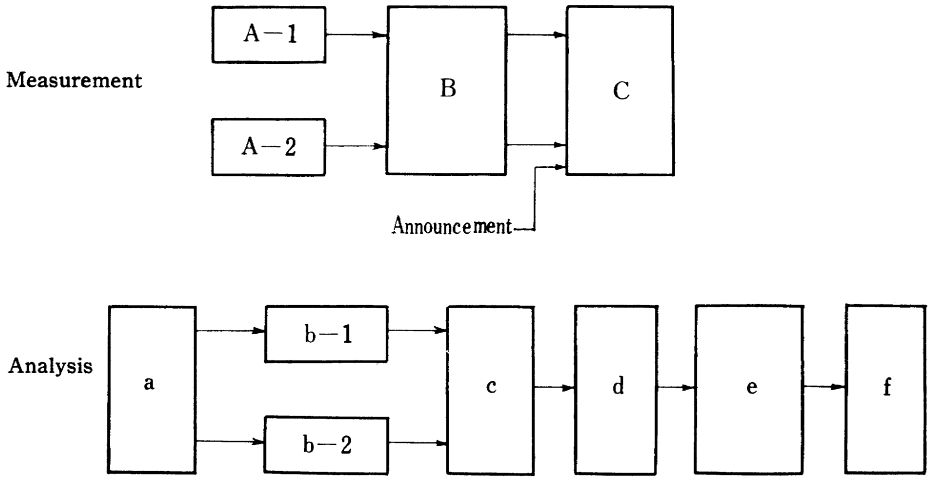

Fig. 3

A-1	:	accellerometer (strain gauge type) mounted on the engine, which indicates $x(t)$.
A-2	:	accellerometer mounted on the body, which indicates $y(t)$.
B	:	strain-meter amplifier.
C	:	3 track magnetic tape recorder (carries out wide modulation of pulse).
a	:	3 track magnetic tape.
b-1, *b*-2	:	band pass filter.
c	:	scanner.
d	:	A–D converter.
e	:	digital computer.
f	:	X–Y plotter.

3. Comparison of the cross spectral method with other methods

For comparison, we refer to two other methods, one by analog computer calculation and the other by a vibration test-stand, and mention about their characteristics.

First, the method by analog computer calculation using values of M, M_E, M_t, K_E, K, K_T, C, C_E in Fig. 2 has the following difficulties in practice.

1) In the case of a heavy vehicle, the measurement of M, M_E requires a large scale equipment and a lot of time.

2) In general, the quantities K_E, K, K_T, C, C_E have non-linear nature and for the convenience of calculation it is necessary to substitute

an equally linearized character for the non-linear characters, but it is often perplexing to equalize the system when we have no knowledge of its actual behavior.

3) The measurement of C, C_E requires special equipments, and as very few measure both the stiffness and damping skillfuly, the measurement by this method requires a lot of people.

Secondly, the measurement of the transfer function by exciting the whole vehicle on a vibration test-stand has the following difficulties.

1) In the case of a heavy vehicle this method requires an exciter of a large capacity and is inevitably very expensive.

2) We do not know to what extent the knowledge of $f(\omega)$ obtained by the vibration test-stand which gives only an input of a simple form to the vehicle pertains to the actual $f(\omega)$ when the system has a random input.

Finally, the time for measurement required by the cross spectral method is shorter than those by the other two. But the method requires a high speed, high memory computer, including A–D converter for the analysis of cross spectra, and that is too expensive for this purpose only. However, we can usually have such a computer for many purposes, so in this case we may put the problem of the equipment cost out of discussion.

As a conclusion, we can say that our method is superior to the other two.

4. Experiments

To analyze the behavior of an engine mounting system by the cross spectral method, the following experiments were made.

First, two accellerometers were mounted, one on the engine cylinder head cover and the other on the front floor board (see Fig. 2), and their outputs were recorded on a magnetic tape (P.W.M.). For this experiment two kinds of engine mounting rubbers were used. Call them A and B respectively. A is an N.B.R. with damping factor=0.3 and B is a natural rubber with damping factor=0.1. The stiffness of A nearly doubles that of B. 10 and 20 km/h were selected as the testing vehicle's speeds, for estimating the influence of the system's non-linearity. In Fig. 4 we show the estimated amplitude gain and phase shift. Evidently, there is a significant difference of characteristics between A and B. Namely, the resonance peak of A which has a larger damping factor than B is lower and broader, and the resonance frequency becomes higher in proportion to the square root of the stiffness ratio of A and B. It could be said that the influence of non-linearity of the system is small, because the amplitude gain and phase have a very slight difference for the vehicle speeds. A very big difference of the estimated phase shifts for A at 15 c.p.s. is nearly 2π and is due to the relatively large sampling error caused by a very low amplitude gain at this frequency. This is explained later by the indication of 100% relative error at this frequency in Fig. 6. It is interesting to note that though the coherencies are rather low the response of the system is fairly stable in the sense that the estimated amplitude gain and phase shift do not show any remarkable change in spite of a significant change in the power spectrum of the input as shown in Fig. 6. This should be interpreted as suggesting the difference between the model in Fig. 2 and the actual system behavior, and, presumably, suggesting the existence of some significant input other than that considered here. Parameters for the estimation were

$$M=1000 \qquad H=100 \qquad K=0$$

and the results by the window W_1 are illustrated.

Table 1. Example of eigenvalues and eigenvectors of an engine mounting (E. Stepp, *SAE Report*, 1961)

C.P.S.	Fore and aft	Vertical	Lateral	Roll	Yaw	Pitch
15.3	1.0	−.060	≈0	≈0	≈0	.041
9.65	−.32	1.0	≈0	≈0	≈0	.036
8.99	≈0	≈0	1.0	.051	−.010	≈0
16.4	≈0	≈0	−1.0	.433	−.106	≈0
12.8	≈0	≈0	− .727	.708	1.0	≈0
10.7	.873	1.0	≈0	≈0	≈0	−.077

5. Conclusion

These results agree fairly well with what was expected before the experiment, and we can get much information by our method which hitherto have not been obtained by other methods. The method will provide a fundamental tool for analysis of the actual car suspension system.

ISUZU MOTOR COMPANY, LTD.

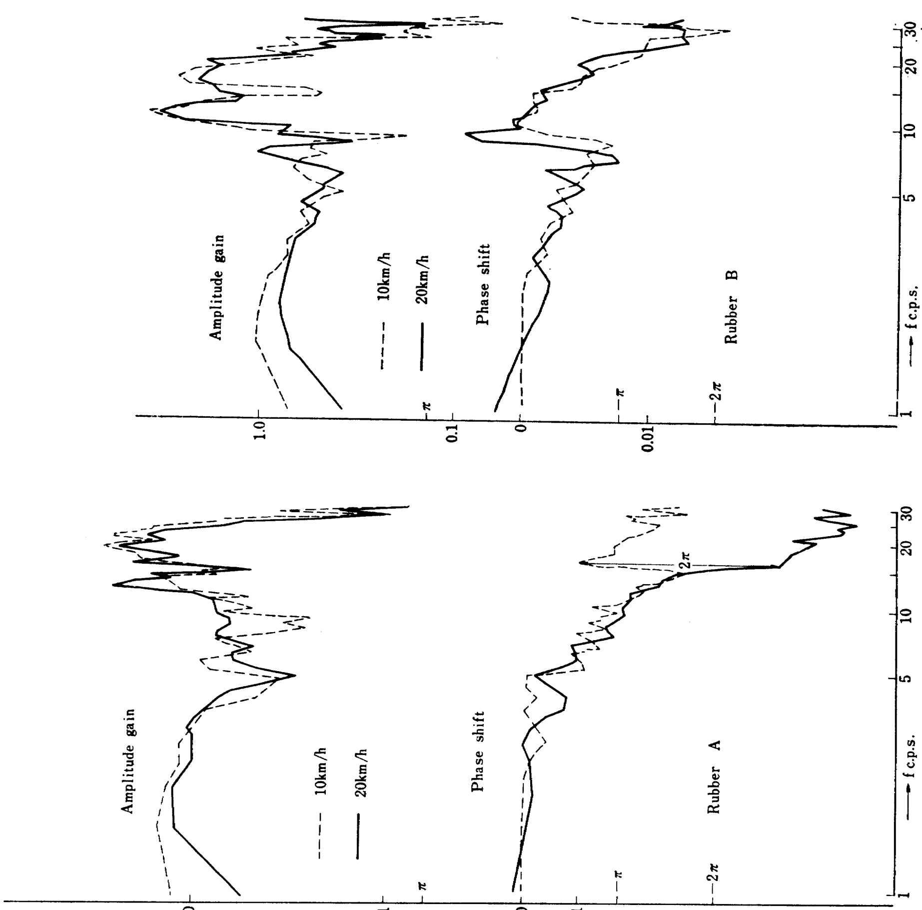
Amplitude gain
10km/h
20km/h
Phase shift
Rubber A
Rubber B
f c.p.s.
1.0
0.1
0
0.01
π
−π
−2π
2π
1
5
10
20
30

Fig. 4

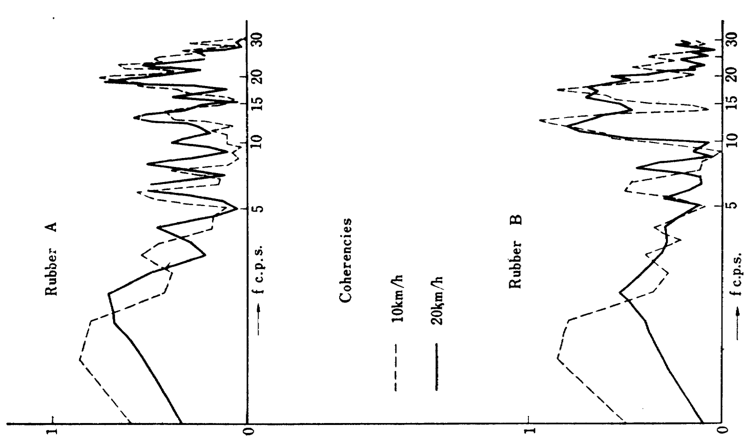
Rubber A
Coherencies
10km/h
20km/h
Rubber B
f c.p.s.
1
0
5
10
15
20
30

Fig. 5

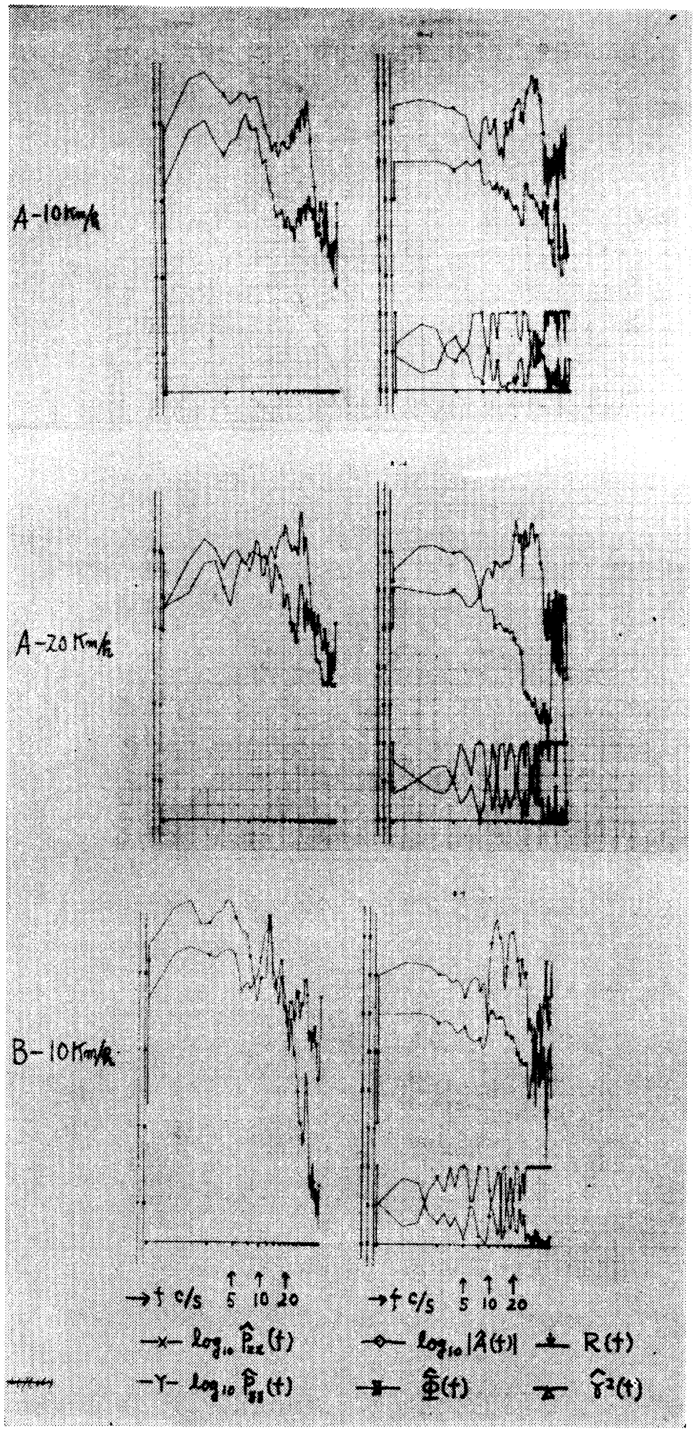
A-10Km/h
A-20Km/h
B-10Km/h
→f c/s 5 10 20
→f c/s 5 10 20
—x— $\log_{10} \hat{P}_{xx}(f)$
—Y— $\log_{10} \hat{P}_{yy}(f)$
$\log_{10}|\hat{A}(f)|$
R(f)
$\hat{\Phi}(f)$
$\hat{\gamma}^2(f)$

Fig. 6

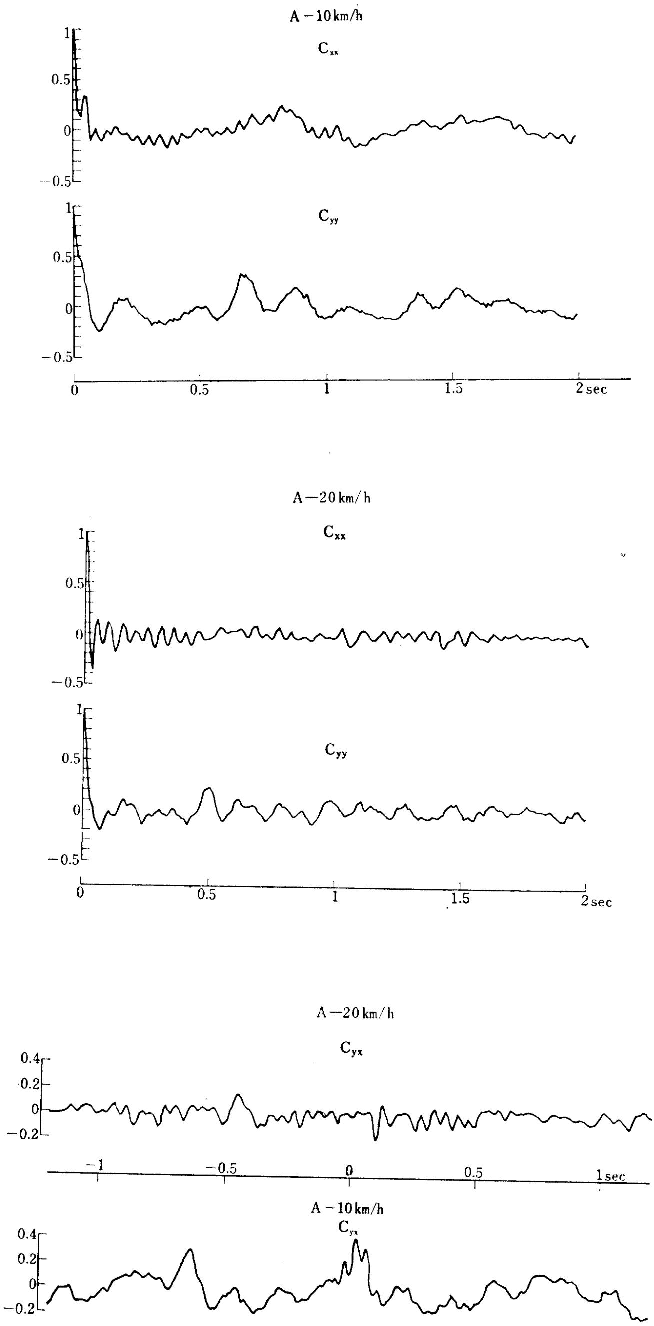
A − 10 km/h
Cxx
1
0.5
0
−0.5
Cyy
1
0.5
0
−0.5
0
0.5
1
1.5
2 sec
A−20 km/h
Cxx
1
0.5
0
−0.5
Cyy
1
0.5
0
−0.5
0
0.5
1
1.5
2 sec
A−20 km/h
Cyx
0.4
0.2
0
−0.2
−1
−0.5
0
0.5
1 sec
A − 10 km/h
Cyx
0.4
0.2
0
−0.2

Fig. 6