2

ANALYSIS OF SHIP OSCILLATIONS IN WAVES

By Yasufumi Yamanouchi

1. Ocean waves

The surface of the ocean which has been expressed by a function of space x, y and time t, $\zeta(x, y, t)$, can be considered as a time series, namely as a sample from a stochastic process. Of course, the irregular moving surface has properties that do not appear to be properly described by linear theory. However, the linear equation of waves obtained through approximation and simplification gives us good results that explain rather nicely the property of wave. Here the record of ocean waves, taken for, say, twenty minutes, is assumed as a sample from a quasi-stationary random process that is approximately Gaussian. Then the spectrum expression is allowed and the wave is expressed as

$$\zeta(x, t)=\int_{-\infty}^{\infty} e^{j(\omega t-kx)}d\xi(\omega) ,$$

where

$$E[d\xi(\omega)d\xi(\omega')]=dS(\omega)\delta(\omega-\omega') ,$$

δ being Dirac's delta function, or following the custom of expression of oceanographers, as

$$\zeta(x, t)=\int_{0}^{\infty}\cos\{\omega t-kx+\varepsilon(\omega)\}\sqrt{[A(\omega)]^2 d\omega} ,$$

$\varepsilon(\omega)$ being the random phase relation between the components. k denotes the wave number $2\pi/\lambda(\omega)=\omega^2/g$, $\lambda(\omega)$ being the wave length.

Here attention should be paid to the definition of spectrum $[A(\omega)]^2$ used customarily by oceanographers. According to it, the spectrum is defined only for the plus side of frequency ω, as is in the expression $\mathfrak{S}(\omega)$ used by Tukey and Rice, but is twice as large as that, because their correlation function is, by definition, twice the correlation function usually defined. Namely,

$$S(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R(\tau)e^{-j\omega\tau}d\tau=\frac{1}{\pi}\int_{0}^{\infty}R(\tau)\cos\omega\tau\, d\tau ,$$

$$\mathfrak{S}(\omega)=\frac{2}{\pi}\int_{0}^{\infty} R(\tau)\cos\omega\tau\, d\tau=\begin{cases}2S(\omega), & \omega>0 ,\\ 0, & \omega<0 ,\end{cases}$$

$$[A(\omega)]^2=\frac{2}{\pi}\int_{0}^{\infty}R'(\tau)\cos\omega\tau\, d\tau=\frac{2}{\pi}\int_{0}^{\infty}2R(\tau)\cos\omega\tau\, d\tau=\begin{cases}4S(\omega), & \omega>0 ,\\ 0, & \omega<0 .\end{cases}$$

Accordingly, the area surrounded by the spectrum on the plus side of ω is $\sigma^2/2$, σ^2, and $2\sigma^2$ respectively according to the definition. Oceanographers use this area $2\sigma^2 \equiv E_0$ as an important parameter to show the dimension of the wave, calling it the Cumulative Energy Spectrum. In theoretical treatment, however, the spectrum $S(\omega)$ defined at $-\infty<\omega<\infty$ is most advisable to use.

Further, thinking of the mechanism of propagation, the concept of the directional spectrum is introduced:

$$\zeta(x, y, t)=\int_0^\infty\int_{-\pi}^{\pi} \cos\left\{\omega t-\frac{\omega^2}{g}(x\cos\theta+y\sin\theta)+\varepsilon(\omega, \theta)\right\}\sqrt{[A(\omega, \theta)]^2 d\omega d\theta}\,.$$

Even in case the wave process is not stationary and contains nonlinear components, the procedure to express it by the correlation function or the spectrum is still useful for showing the character of waves.

When the wind blows continuously over the surface of water, the wave is generated, and after a certain duration of time and at the area with a sufficiently large fetch of waves arround it, that reaches to a saturated condition, the so-called fully arisen sea. During these ten or fifteen years, many oceanographers like Neumann, Pierson, Roll and Fisher, Darbyshire, Bretschneider, Barling, Moskowitz and Pierson, Miles and Phillips have proposed semiempirical or theoretical expressions for the spectrum of wind generated seas. At present, none of them is especially authorized. However, all of them have a similar pattern, that is, they are all almost vertical at the lower frequency side and proportional to c/ω^n at higher frequencies, n ranging from 4 to 8 according to the proposer (see Fig. 1). For example, the spectrum proposed by Neumann is

$$[A(\omega)]^2=\frac{C\rho g^3\pi^3}{\omega^6}e^{-\frac{2g^2}{\omega^2U^2}}\,, \quad U:\ \text{wave velocity}$$

and Pierson's modified directional spectrum is

$$[A(\omega, \theta)]^2=\frac{C'}{\omega^6}e^{-\frac{2g^2}{\omega^2U^2}}(\cos\theta)^2\,.$$

While Longuet-Higgins insists that

$$[A(\omega, \theta)]^2 \propto \left(\cos\frac{\theta}{2}\right)^{2s},$$

where s is $0.1\sim7$ and decreases by the increase of ω, namely the spectrum becomes broad in the range of high frequencies.

The wave propagated to the distant place from the generating zone is called the swell. Usually that is a rather long regular wave with a long crest line, because the short waves decay down on the way. Accordingly, on one point of the ocean, usually there exists more than two wave systems, which causes a rather complicated form of spectrum with more than two peaks.

2. Character of response of ship oscillations

Neglecting the effect of the wind and the oscillatory force due to the propeller working in waves, the ship oscillation in waves around her equilibrium position is expressed by the equation of motion of a rigid body. The axes are taken along her principal axes of inertia (see Fig. 2).

$$
(2.1)\quad \begin{cases} m\ddot{x}=\mathfrak{F}_1(\ddot{x},\ \dot{x},\ x\ ;\ \ddot{y},\ \dot{y},\ y\ ;\ \ddot{z},\ \dot{z},\ z\ ;\ \ddot{\phi},\ \dot{\phi},\ \phi\ ;\ \ddot{\theta},\ \dot{\theta},\ \theta\ ;\ \ddot{\psi},\ \dot{\psi},\ \psi\ ;\ \zeta) \\ \cdots\cdots \\ I_x\ddot{\phi}-(I_y-I_z)\dot{\theta}\dot{\psi}=\mathfrak{M}_1(\ddot{x},\ \dot{x},\ x\ ;\ \ddot{y},\ \dot{y},\ y\ ;\ \ddot{z},\ \dot{z},\ z\ ;\ \ddot{\phi},\ \dot{\phi},\ \phi\ ;\ \ddot{\theta},\ \dot{\theta},\ \theta\ ; \\ \qquad\qquad \ddot{\psi},\ \dot{\psi},\ \psi;\ \zeta) \\ \cdots\cdots \end{cases}
$$

The fluid force $\mathfrak{F}_i$ and moment $\mathfrak{M}_i$ can be assumed to be linear combinations of forces due to the ship motions in still water and the force due to the coming waves. The ship is regarded as a thin body symmetric to xz-plane, and the cross coupling hydrodynamic forces between the symmetric motions as surging, pitching and heaving, and the anti-symmetric motions as rolling, swaying and yawing, have been found to be very small. As the results, the equation of the symmetric motions in simple harmonic waves, for example, becomes

$$
(2.2)\quad \begin{cases} m\ddot{x}+a_{11}\ddot{x}+a_{12}\dot{x}+a_{13}x+a_{14}\ddot{\theta}+a_{15}\dot{\theta}+a_{16}\theta+a_{17}\ddot{z}+a_{18}\dot{z}+a_{19}z=E_1e^{j\omega t}\ , \\ I_y\ddot{\theta}+a_{21}\ddot{x}+a_{22}\dot{x}+a_{23}x+a_{24}\ddot{\theta}+a_{25}\dot{\theta}+a_{26}\theta+a_{27}\ddot{z}+a_{28}\dot{z}+a_{29}z\ =E_2e^{j\omega t}\ , \\ m\ddot{z}+a_{31}\ddot{x}+a_{32}\dot{x}+a_{33}x+a_{34}\ddot{\theta}+a_{35}\dot{\theta}+a_{36}\theta+a_{37}\ddot{z}+a_{38}\dot{z}+a_{39}z=E_3e^{j\omega t}\ . \end{cases}
$$

Here the right-hand side is the forces or moments due to the external waves, and can be put in the form

$$
(2.3)\qquad E_i(\omega)e^{j\omega t}=L_i(\omega)\zeta(\omega)e^{j\omega t}\ ,
$$

multiplying the complex coefficients $L_i(\omega)$ by the simple harmonic waves expressed by $\zeta(t)=\zeta(\omega)e^{j\omega t}$. Then, after a long period of time, the motions will also be simple harmonic,

$$
(2.4)\qquad x(t)=X(\omega)e^{j\omega t},\ \ \theta(t)=\Theta(\omega)e^{j\omega t},\ \ z(t)=Z(\omega)e^{j\omega t}\ ,
$$

$X(\omega)$, $\Theta(\omega)$, $Z(\omega)$ being the complex amplitudes of respective oscillations.

Putting the frequency response as

$$X(\omega)/\zeta(\omega)=H_{x\zeta}(\omega),\ \Theta(\omega)/\zeta(\omega)=H_{\theta\zeta}(\omega),\ Z(\omega)/\zeta(\omega)=H_{z\zeta}(\omega)\ , \tag{2.5}$$

the equation of motion can be modified into the equation of frequency response function $H(\omega)$ as

$$\begin{bmatrix} P_1(\omega) & Q_1(\omega) & R_1(\omega) \\ P_2(\omega) & Q_2(\omega) & R_2(\omega) \\ P_3(\omega) & Q_3(\omega) & R_3(\omega) \end{bmatrix} \begin{bmatrix} H_{x\zeta}(\omega) \\ H_{\theta\zeta}(\omega) \\ H_{z\zeta}(\omega) \end{bmatrix} = \begin{bmatrix} L_1(\omega) \\ L_2(\omega) \\ L_3(\omega) \end{bmatrix}, \tag{2.6}$$

where, according to the eq. (2.2),

$$\left\{ \begin{array}{l} P_1(\omega)=[-(m+a_{11})\omega^2+j\omega a_{12}+a_{13}] \\ Q_1(\omega)=[-a_{14}\omega^2+j\omega a_{15}+a_{16}] \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \end{array} \right. \tag{2.7}$$

From eq. (2.6), $H_{x\zeta}(\omega)$, $H_{\theta\zeta}(\omega)$ and $H_{z\zeta}(\omega)$ can be solved formally. As is well known,

$$\begin{aligned} H(\omega) &= |H(\omega)|e^{j\delta(\omega)}=|H(\omega)|\cos\delta(\omega)+j|H(\omega)|\sin\delta(\omega) \\ &=c(\omega)+jq(\omega)=\int_{-\infty}^{\infty}h(\tau)e^{-j\omega\tau}d\tau \end{aligned} \tag{2.8}$$

where $h(\tau)$ is the impulse response function.

The above mentioned equations of motion are for the ship in sinusoidal long crested waves. As all of the coefficients a_{11}, a_{12}, $\cdots$, a_{39} ; E_1, E_2, E_3 in eq. (2.2) are not constant, but are dependent on the frequency of encounter with the wave, this equation can not be used when the external wave is composed of more than two element waves of different frequencies. If the wave $\zeta(t)$ is irregular, and is assumed to be the sum of infinite component waves as

$$\zeta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\zeta(\omega)e^{j\omega t}d\omega\ , \tag{2.9}$$

the existence of the Fourier transform being assumed, then the linear response to this wave is

$$\begin{aligned} &x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega t}d\omega,\ \theta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Theta(\omega)e^{j\omega t}d\omega\ , \\ &z(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}Z(\omega)e^{j\omega t}d\omega\ . \end{aligned} \tag{2.10}$$

Namely, the response is also the sum of responses to each component waves.

Then the equation of motion for this case is

$$(2.11)\quad \frac{1}{2\pi}\int_{-\infty}^{\infty} P_i(\omega)X(\omega)e^{j\omega t}d\omega+\frac{1}{2\pi}\int_{-\infty}^{\infty} Q_i(\omega)\Theta(\omega)e^{j\omega t}d\omega+\frac{1}{2\pi}\int_{-\infty}^{\infty} R_i(\omega)Z(\omega)e^{j\omega t}d\omega$$
$$=\frac{1}{2\pi}\int_{-\infty}^{\infty} L_i(\omega)\zeta(\omega)e^{j\omega t}d\omega \qquad \text{for } i=1,\ 2,\ 3\,.$$

Further, assuming the existence of the Fourier transform

$$(2.12)\quad \begin{aligned} &\int_{-\infty}^{\infty} P_i(\omega)e^{j\omega t}d\omega=p_i(t), \quad \int_{-\infty}^{\infty} Q_i(\omega)e^{j\omega t}d\omega=q_i(t)\,, \\ &\int_{-\infty}^{\infty} R_i(\omega)e^{j\omega t}d\omega=r_i(t), \quad \int_{-\infty}^{\infty} L_i(\omega)e^{j\omega t}d\omega=l_i(t)\,, \end{aligned}$$

the eq. (2.11) is expressed in the form

$$(2.13)\quad \int_{-\infty}^{\infty} p_i(\tau)x(t-\tau)d\tau+\int_{-\infty}^{\infty} q_i(\tau)\theta(t-\tau)d\tau+\int_{-\infty}^{\infty} r_i(\tau)z(t-\tau)d\tau=\int_{-\infty}^{\infty} l_i(\tau)\zeta(t-\tau)d\tau,$$

or, using the expression of convolution,

$$(2.14)\quad p_i(t)*x(t)+q_i(t)*\theta(t)+r_i(t)*z(t)=l_i(t)*\zeta(t) \qquad \text{for } i=1,\ 2,\ 3.$$

These simultaneous eq. (2.14) have apparently the same form with eq. (2.6). However, as each term is a convolution, they can not be solved formally. If $P_i(\omega)$, $Q_i(\omega)$, $R_i(\omega)$, $L_i(\omega)$ are expressed by polynomials of ω, as is usually the case, for example,

$$P_1(\omega)=c_0\omega^n+c_1\omega^{n-1}+\cdots+c_n=\sum_{i=0}^{n}c_i\omega^{n-i}\,,$$

then

$$p_1(\tau)=\int_{-\infty}^{\infty}\sum_{i=0}^{n}c_i\omega^{n-i}e^{j\omega\tau}d\omega=\sum_{i=0}^{n}c_i\int_{-\infty}^{\infty}\omega^{n-i}e^{j\omega\tau}d\omega=\sum_{i=0}^{n}c_i(j)^{n-i}\frac{d^{n-i}}{dt^{n-i}}\delta(\tau)\,,$$

and the first term of eq. (2.13) or (2.14) is

$$p_1(t)*x(t)=\sum_{i=0}^{n}c_i(j)^{n-i}\int_{-\infty}^{\infty}\frac{d^{n-i}}{dt^{n-i}}\delta(\tau)x(t-\tau)d\tau=\sum_{i=0}^{n}c_i(j)^{n-i}(-1)^{n-i}\frac{d^{n-i}x(t)}{dt^{n-i}}\,.$$

This shows that, in this case the equation of motion is expressed by the ordinary differential equation of $x(t)$, $\theta(t)$, $z(t)$, of an order higher than 2 and with constant coefficients c_i. Accordingly, this equation will be solved formally as eq. (2.6) if we want. It is interesting to note that, for example, the first term of eq. (2.13) can also be converted into the form

$$\int_{-\infty}^{\infty} b_{11}(\tau)\ddot{x}(t-\tau)d\tau+\int_{-\infty}^{\infty} b_{12}(\tau)\dot{x}(t-\tau)d\tau+\int_{-\infty}^{\infty} b_{13}(\tau)x(t-\tau)d\tau$$

and shows that not only the present value, but also the history of the acceleration, velocity and displacement are necessary for treating this kind of motion.

Anyway, one of the characteristics of ship oscillation is that all the coefficients of eq. (2.2) are functions of frequency, and as a result, the expression of response by the concept of frequency response is much more understandable and convenient than solving the higher order differential equations of motion computing all the coefficients at various frequencies theoretically or empirically. Here we can say that this case of ship oscillation is a good example where the frequency response plays an important role in analysis of the response.

3. Analysis of ship response in waves

When the ship encounters the wave, the encounter frequency ω_e is given by (see Fig. 3)

$$\omega_e = \omega - kv\cos\chi = \omega - \frac{\omega^2}{g}v\cos\chi\,.$$

If the spectrum of long crested waves at a fixed point in space is given by an oceanographer, that should at first be converted into the spectrum in terms of encounter frequency (see Fig. 4).

$$[A(\omega_e)]^2 = [A(\omega)]^2\,|J|\,,\quad |J| = \left|\frac{\partial\omega}{\partial\omega_e}\right|\,.$$

When the encountering waves are measured directly on board the ship, the record can, of course, be used as the input. From the spectra of ship oscillations and waves, the frequency response of that oscillation is calculated. For example, corresponding to

$$z(t) = \int_{-\infty}^{\infty} h(\tau)\zeta(t-\tau)d\tau$$

we have

$$\begin{cases} R_{z\zeta}(\tau) = \int_{-\infty}^{\infty} h_{z\zeta}(\alpha)\cdot R_{\zeta\zeta}(\tau-\alpha)d\alpha\,, & S_{z\zeta}(\omega) = H(\omega)S_{\zeta\zeta}(\omega)\,, \\ R_{zz}(\tau) = \int_{-\infty}^{\infty} h_{z\zeta}(\alpha)\overline{h_{z\zeta}(\beta)}R_{\zeta\zeta}(\tau-\alpha-\beta)d\alpha d\beta\,, & S_{zz}(\omega) = |H(\omega)|^2\cdot S_{\zeta\zeta}(\omega)\,. \end{cases}$$

Conversely, if we know the frequency response character and the spectrum of waves, the spectrum of response oscillation can be estimated. The spectrum of response oscillation is used to estimate various expected values of oscillation and to predict the character of behavior of the ship. Namely, for example, getting the parameter ϵ that shows the effective band-width of the spectrum as

$$\epsilon^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4}\,,\quad m_i = \int_{-\infty}^{\infty}\omega^i S(\omega)d\omega\,,$$

the expected highest values of $1/\varphi$ or the expected maximum, the so-called extreme value of the independent samples (number N) of oscillation, are estimated as multiples of $\sqrt{E_0}$, where E_0 is related to the area

Table 1.

	$\epsilon=0$	$\epsilon=\sqrt{2/3}$
$\xi_{(ave.)}/\sqrt{E_0}$	0.886	0.511
$\xi_{(1/3)}/\sqrt{E_0}$	1.416	1.200
$\xi_{(1/10)}/\sqrt{E_0}$	1.800	1.643
$\xi_{max}(N=50)/\sqrt{E_0}$	2.124	1.991
$\xi_{max}(N=100)/\sqrt{E_0}$	2.280	2.157
$\xi_{max}(N=1000)/\sqrt{E_0}$	2.738	2.636

of the spectrum by $E_0=2m_0$. In Table 1, multiples of $\sqrt{E_0}$ are shown for two cases $\epsilon=0$ and $\sqrt{2/3}$, corresponding to the ideally narrow band spectrum and to Neumann's spectrum of wind generated waves. The former corresponds to the rolling of ship in arbitrary waves which has a very sharp and narrow band response, and the latter corresponds to the heaving of ship in the fully arisen sea which has a very broad and flat response character.

4. Examples of analysis

A lot of continuous measurements of oscillations as well as the other kind of response of ship in waves, like the stress induced on her hull, were performed on board the ship running straight on the line. To our regret, however, the simultaneous wave measurement was impossible because a satisfactory wave measuring instrument was not yet available. Here in this section, as an example, the result of a model experiment in an experimental water tank where the simultaneous wave measurement is possible will be analyzed.

The model ship has the scale of about 1.5 m and the so-called Todd series 60, block coefficient 0.60, and is equipped with all appendages like bilge keels, a rudder and a propeller screw. This example is the case when the ship has no advance speed and is rolling in the beam sea. The irregular wave system was produced by a plunger type wave machine installed on one side of the tank. The rolling, pitching, yawing, surging, swaying and heaving were recorded together with the wave height at distance d from the c.g. of the moving model. The records of rolling, heaving and wave height were analyzed and the response of roll and heave to the wave were calculated (see Fig. 5). The particulars used in the analysis are as follows:

Sampling interval	$\Delta t=0.3$ sec,
Number of data	$M=698$,
Windows used	W_1, W_2 and W_3,
Max. number of lag used	$h=90$.

The auto-correlation coefficients of wave, roll and heave, and also the cross-correlation coefficient of wave-roll and wave-heave are shown in Figs. 6 and 7. The correlation were computed at first up to lag $H=199$, and were decided to adopt up to $h=90$, in the analysis. The wave-roll cross-correlation was first computed at small values of h, and finding the peak of it at around $h=9$, the origin of cross-correlation was shifted to $h=9$, namely the time axis was shifted $T_p=S\Delta t=9\Delta t$. This means that the response of roll $9\Delta t$ lagging to the wave was taken as the corresponding response, and in the new computation $9\Delta t$ lag was expressed as 0 lag. The power spectra of wave, roll and heave computed using the filter W_1 are shown in Figs. 8, 9 and 10. We can see high coherencies in the frequency range where amplitude gains are high. The same computation were tried with windows W_2 and W_3, and naturally gave a little steeper peak and deeper valley as the suffix of window increases. However, the difference is rather small.

The frequency responses of roll to wave and heave to wave are shown in Figs. 11 and 12. The effect of difference of windows is less in this result and checks the fact that the max. lag $h=90$ was sufficiently long for this case. As was already reported, the shift of response, in other words, the shift of origin for cross-correlation by T_p as is shown in Fig. 13 is very effective to make smaller the bias in computing the cross-spectrum and as a result improves the coherency. That is especially clear in case when the maximum lag $T_m=h\Delta t$ is small compared with T_p, when T_p is a large fraction of T_m, but when T_p is a small fraction of T_m as is the case in this example, this improvement does not appear so clearly. This is quite natural if we think of the fact that to take the lagged response as was mentioned means to take the most closely related response in correlating the output to the input. The $9\Delta t$ shift comes from the lag of roll to wave height itself and also by the lead of wave height measurement to the measurement of roll. Of course the phase should be corrected of this shift by ωT_p at frequency ω.

The gain of roll was shown in Fig. 11 by the ratio of roll angle ϕ to the wave slope γ. For that purpose, the computed frequency response of the roll angle to the wave height ζ was modified as follows.

$$H_{\phi\gamma}(\omega)=H_{\phi\zeta}(\omega)\frac{\zeta(\omega)}{\gamma(\omega)}=H_{\phi\zeta}\frac{g}{\omega^2}e^{-j\frac{\pi}{2}} .$$

This includes the phase correction by $\pi/2$ by the lead of the wave slope to the height. Together with the phase correction by the lead of wave measurement by distance d, and by the shift of response in computation, all the correction is

$$-\omega T_p+2\pi\frac{d}{\lambda}-\frac{\pi}{2} .$$

The phase shift is shown in Figs. 9 and 10 for roll and heave respectively.

From the frequency response $H(\omega)$ for roll to wave height, the impulse response was computed as in Fig. 14. The response of roll was shifted by $9\Delta t$, accordingly the real 0 for time scale for the impulse response is on $-9\Delta t$. This impulse response is not so beautiful, and shows that the frequency response $H(\omega)$ used, is still distorted by various noises.

5. Conclusion

The ship oscillations in waves have at least 6 degrees of freedom. There are a lot of other ship responses in waves as the propeller torque and thrust fluctuations, stress variations, or the change of these performances by the maneuvering of the rudder through the auto-pilot system. In order to make clear these actual characters in waves, finding the mechanism of mutual interference of these elements of responses, more advanced analyzing techniques are needed. The author sincerely wishes to achieve this developments through the active co-operation of statisticians, oceanographers and naval architects.

SHIP RESEARCH INSTITUTE

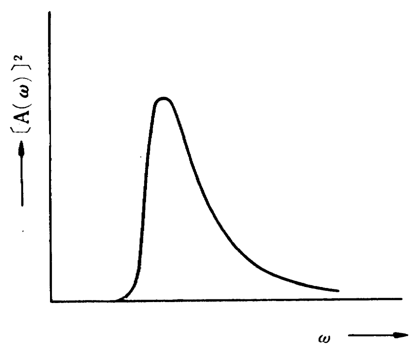

Fig. 1 Spectrum of wind generated sea

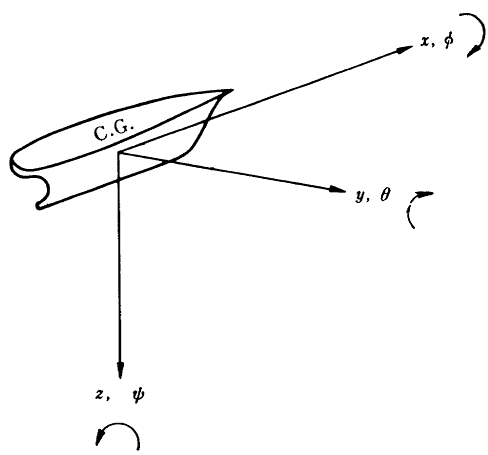

Fig. 2 Co-ordinate for ship motion

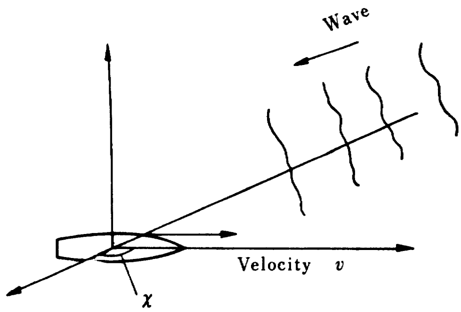

Fig. 3 Encounter of waves

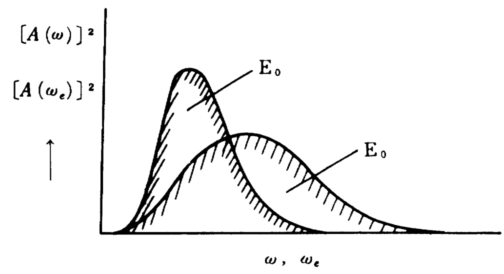

Fig. 4 Modification of spectrum

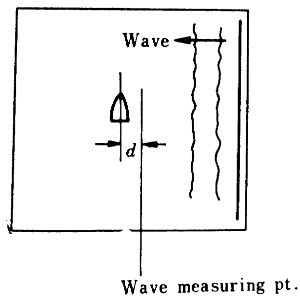

Fig. 5 Test in the tank

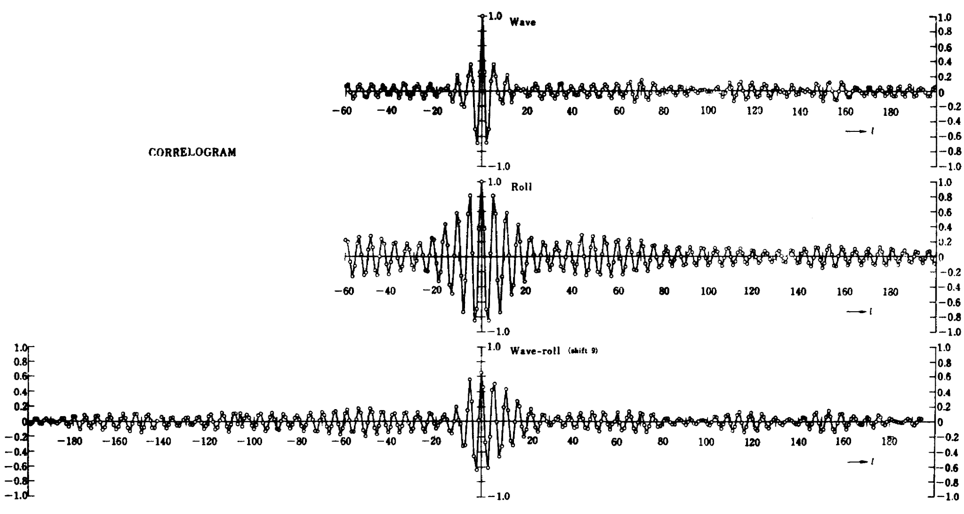

Fig. 6

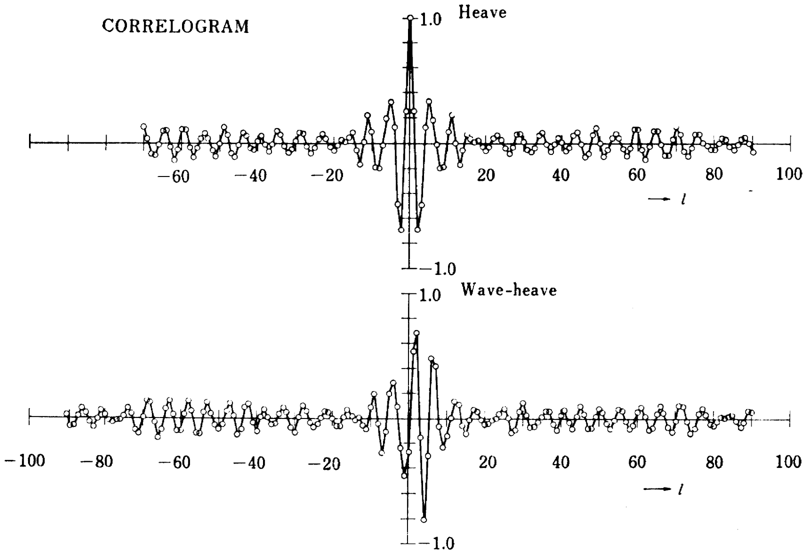

Fig. 7

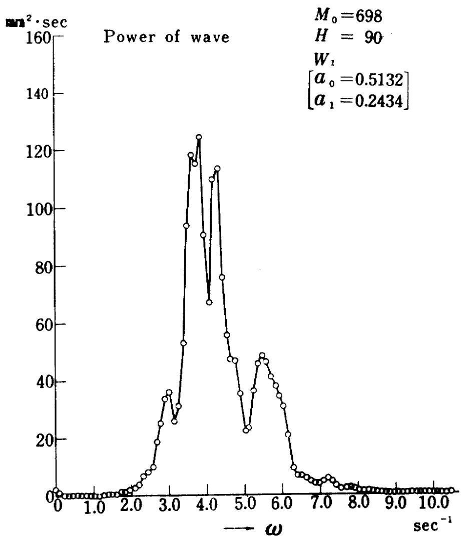

Fig. 8

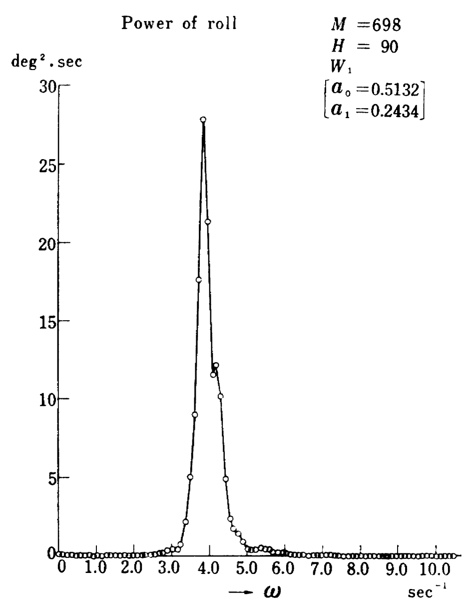

Fig. 9

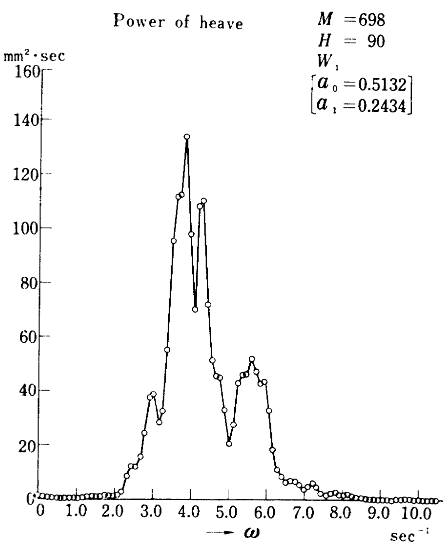

Fig. 10

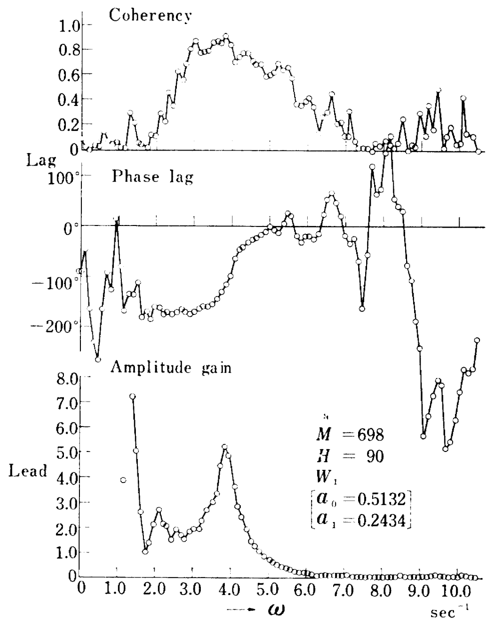

Fig. 11 Response of roll to wave slope

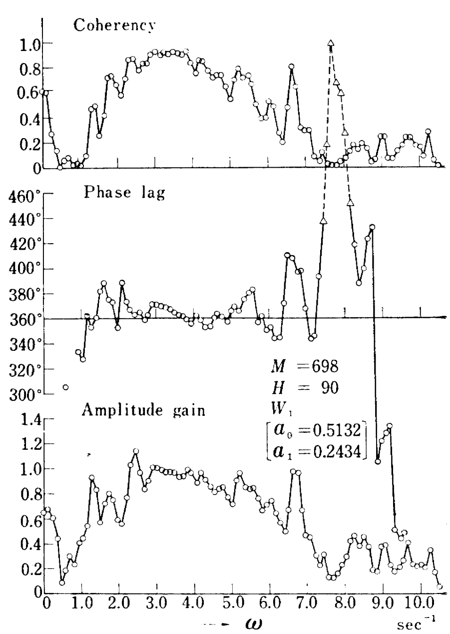

Fig. 12 Response of heave to wave height

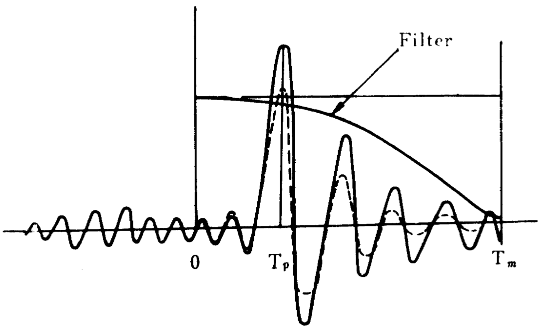

Fig. 13 Effect of filter for cross-correlation

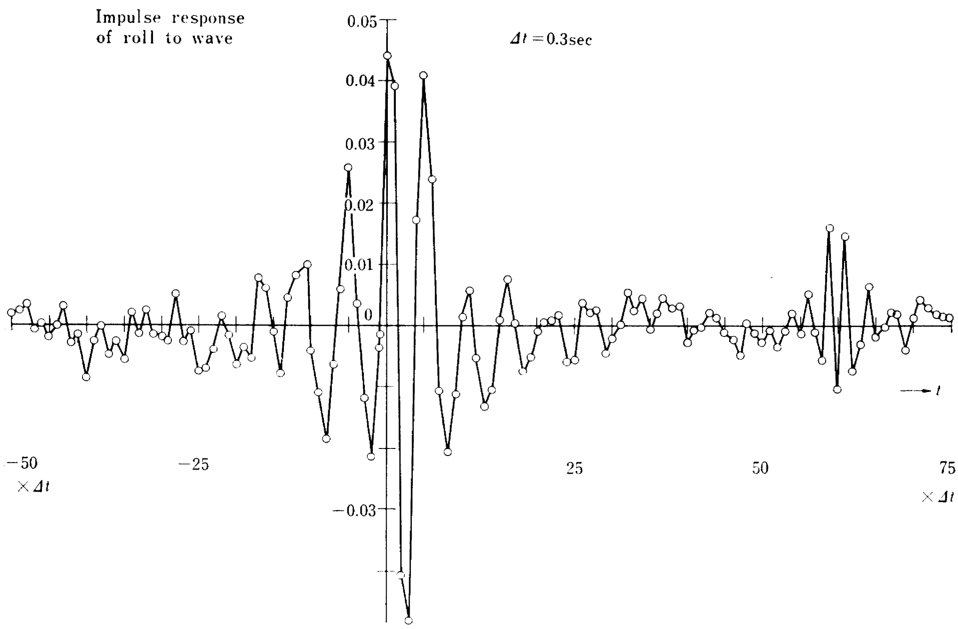

Fig. 14