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STATISTICAL MEASUREMENT OF FREQUENCY RESPONSE FUNCTION

BY H. AKAIKE

1. Introduction

In this paper we deal with a statistical method of estimation of the frequency response function, which was applied to the obtaining of the numerical results reported in other papers of this supplement. In our research project to develop the method, the present author was in charge of processing all the numerical data necessitated for the investigation, and some general observations experienced therein were combined with the basic results in the former paper by Y. Yamanouchi and the present author [1] to give the method a form, stated in this paper, suitable for practical applications. Some conclusions obtained from the practical applications of the method are also described.

2. Glossary of symbols

Throughout this paper we shall use the symbols given in the following glossary. The same symbols used without explicit definition in the other papers of this supplement have the same meanings as in this glossary.

f : frequency.

t : time.

Δt : time interval between adjacent data values.

$x(t)$: input to the system, assumed to be a weakly stationary stochastic process for non-linear system.

$y(t)$: output of the system, under the input $x(t)$ and usually contaminated with noise.

$x(n)$: $x(n\Delta t)$.

$y(n)$: $y(n\Delta t)$.

$A(f)$: frequency response function of the system, when the system is linear and time-invariant; otherwise, that of the corresponding linearized system.

$|A(f)|$: amplitude gain.

$\Phi(f)$: Arg $A(f)$, phase shift defined by

$$A(f)=|A(f)|\exp(i\Phi(f)) \qquad (i^2=-1).$$

W_1, W_2, W_3: spectral windows to be defined in the following section.
H: maximum number of lags of correlations computed.
h: number of lags of correlations used for the computation of the estimates.
K: amount of shift of the data window for the computation of the sample cross spectrum.
M: number of data used, $M\Delta t$ = total length of observation.
S: amount of time shift introduced in the original recording, $y(t+S\Delta t)$ is recorded in place of $y(t)$.*

3. Data processing scheme

Analysis of record proceeds as follows.

Selection of parameters

P–1. Δt: Powers above the frequency $1/2\Delta t$ of $x(t)$ and $y(t)$ should be at sufficiently low level compared with those in the frequency range of the present concern.

excessively small Δt ⟶ loss of computing time.
excessively large Δt ⟶ bias due to aliasing occurs, usually in a higher frequency range.

P–2. h: h should be chosen so as to satisfy

$$h\Delta t \geqq 1/B,$$

where B is the band-width of the peak of $|A(f)|$ of main concern as defined by Fig. 1.

excessively large h ⟶ increase of sample variance.
excessively small h ⟶ bias occurs, usually giving under-estimate of $|A(f)|$.

Also confer P–3, the definition of K.

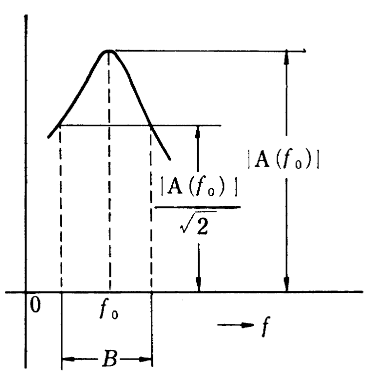

Fig. 1

P–3. K: To compensate for the bias due to phase shift, K should be chosen so that we have

* The value of S satisfying $|S|<1$ was often used in our analysis, which was introduced by the lack of a sample hold circuit in the A/D conversion system. It is not explicitly stated in the papers of this supplement, though the necessary compensation was made during computation.

$$\left| K\Delta t - \left(-\left(\frac{1}{2\pi}\right)\left(\frac{d\Phi(f)}{df}\right)\right)\right| \leqq \begin{cases} 0.15\, h\Delta t & \text{for } W_1 \\ 0.30\, h\Delta t & \text{for } W_2 \\ 0.40\, h\Delta t & \text{for } W_3\,, \end{cases}$$

otherwise, $|A(f)|$ will be under-estimated by more than 5%. K should also be made so as to satisfy

$$|K\Delta t| \leqq 0.05\, M\Delta t\,,$$

otherwise, adopt $y(t+S\Delta t)$ as $y(t)$ where $S=K$.

P–4. M: As a somewhat conservative estimate

$$M\Delta t \geqq 5\, h\Delta t\,.$$

If possible, $M\Delta t \geqq 10\, h\Delta t$ will be more desirable. Confer the comment M–1.

Computing scheme

C–1. C_{xx}: Compute the following

$$C_{xx}(l) = \frac{1}{M}\sum_{n=1}^{M-l} \tilde{x}(l+n)\tilde{x}(n) \qquad l=0,\ 1,\ 2,\ \cdots,\ h$$

where

$$\tilde{x}(n) = x(n) - \bar{x}\,,$$
$$\bar{x} = \frac{1}{M}\sum_{n=1}^{M} x(n)\,.$$

C–2. C_{yy}: Use y in place of x in the above formula for C_{xx}.

C–3. C_{yx}: Compute the following

$$C_{yx}(l) = \frac{1}{M}\sum_{n=1}^{M-l} \tilde{y}(l+n)\tilde{x}(n) \qquad (l \geqq 0)$$
$$= \frac{1}{M}\sum_{n=-l}^{M} \tilde{y}(l+n)\tilde{x}(n) \qquad (l<0)$$

$$l = -h_1,\ -h_1+1,\ \cdots,\ -1,\ 0,\ 1,\ \cdots,\ h_2-1,\ h_2$$

where h_1 and h_2 are to satisfy

$$-h_1 \leqq -h+K\,,$$
$$h_2 \geqq h+K\,.$$

The last requirements suggest that, in the pilot estimation, C_{xx}, C_{yy} and C_{yx} should be computed for somewhat longer lags of l than would be guessed from the above definitions of h and K. This is the procedure adopted in the flow diagram for the estimation procedure in Fig. 2, where a sufficiently large value H takes place of h, h_1 and h_2 in C-1, -2 and -3.

C-4. $\bar{P}_{xx}$: Compute the following

$$\bar{P}_{xx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\Delta t\left\{C_{xx}(0)+2\sum_{l=1}^{h-1}C_{xx}(l)\cos\left(2\pi\frac{r}{2h}l\right)+(-1)^r C_{xx}(h)\right\}$$

$$r=0,\ 1,\ 2,\ \cdots,\ h\ .$$

C-5. $\bar{P}_{yy}$: Use y in place of x in the above formula for $\bar{P}_{xx}$.

C-6. $\bar{P}_{yx.K}$: Compute the following

$$\bar{c}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\Delta t\left\{C^{+}_{yx.K}(0)+2\sum_{l=1}^{h-1}C^{+}_{yx.K}(l)\cos\left(2\pi\frac{r}{2h}l\right)\right.$$

$$\left.+(-1)^r C^{+}_{yx.K}(h)\right\}$$

where

$$C^{+}_{yx.K}(l)=\frac{1}{2}\left\{C_{yx}(K+l)+C_{yx}(K-l)\right\}$$

and

$$\bar{s}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=-\Delta t\left\{2\sum_{l=1}^{h-1}C^{-}_{yx.K}(l)\sin\left(2\pi\frac{r}{2h}l\right)\right\}$$

where

$$C^{-}_{yx.K}(l)=\frac{1}{2}\left\{C_{yx}(K+l)-C_{yx}(K-l)\right\}\ .$$

Then $\bar{P}_{yx.K}$ is given by

$$\bar{P}_{yx.K}(f)=\bar{c}_{yx.K}(f)+i\bar{s}_{yx.K}(f) \qquad \text{where } i^2=-1.$$

C-7. $\hat{P}_{xx}$: Smooth $\bar{P}_{xx}$ using the window $\{a_n\}$ given in C-15.

$$\hat{P}_{xx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\sum_{n=-k}^{k}a_n\bar{P}_{xx}\left(\frac{r-n}{h}\frac{1}{2\Delta t}\right) \qquad r=0,\ 1,\ 2,\ \cdots,\ h$$

where

$$\bar{P}_{xx}\left(\frac{-r}{h}\frac{1}{2\Delta t}\right)=\bar{P}_{xx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$

$$\bar{P}_{xx}\left(\frac{h+r}{h}\frac{1}{2\Delta t}\right)=\bar{P}_{xx}\left(\frac{h-r}{h}\frac{1}{2\Delta t}\right).$$

C–8. $\hat{P}_{yy}$: Smooth $\bar{P}_{yy}$ to get $\hat{P}_{yy}$ in the same way as in C–7.

C–9. $\hat{P}_{yx.K}$: Smooth $\bar{c}_{yx.K}$ and $\bar{s}_{yx.K}$ by $\{a_n\}$ in the following way to get $\hat{c}_{yx.K}$ and $\hat{s}_{yx.K}$.

$$\hat{c}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\sum_{n=-k}^{k}a_n\bar{c}_{yx.K}\left(\frac{r-n}{h}\frac{1}{2\Delta t}\right) \qquad r=0,\,1,\,2,\,\cdots,\,h$$

where

$$\bar{c}_{yx.K}\left(\frac{-r}{h}\frac{1}{2\Delta t}\right)=\bar{c}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$

$$\bar{c}_{yx.K}\left(\frac{h+r}{h}\frac{1}{2\Delta t}\right)=\bar{c}_{yx.K}\left(\frac{h-r}{h}\frac{1}{2\Delta t}\right).$$

$$\hat{s}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\sum_{n=-k}^{k}a_n\bar{s}_{yx.K}\left(\frac{r-n}{h}\frac{1}{2\Delta t}\right) \qquad r=0,\,1,\,2,\,\cdots,\,h$$

where

$$\bar{s}_{yx.K}\left(\frac{-r}{h}\frac{1}{2\Delta t}\right)=-\bar{s}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$

$$\bar{s}_{yx.K}\left(\frac{h+r}{h}\frac{1}{2\Delta t}\right)=-\bar{s}_{yx.K}\left(\frac{h-r}{h}\frac{1}{2\Delta t}\right).$$

$\hat{P}_{yx.K}$ is given by

$$\hat{P}_{yx.K}(f)=\hat{c}_{yx.K}(f)+i\hat{s}_{yx.K}(f).$$

C–10. $\hat{P}_{yx}$: Compensate for the time shift $K\Delta t$ to get $\hat{P}_{yx}$ from $\hat{P}_{yx.K}$.

$$\hat{P}_{yx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\hat{c}_{yx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)+i\hat{s}_{yx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$

where

$$\hat{c}_{yx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\cos\left(2\pi\frac{r}{2h}K\right)\hat{c}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$

$$+\sin\left(2\pi\frac{r}{2h}K\right)\hat{s}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$

$$\hat{s}_{yx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\cos\left(2\pi\frac{r}{2h}K\right)\hat{s}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$
$$-\sin\left(2\pi\frac{r}{2h}K\right)\hat{c}_{yx.K}\left(\frac{r}{h}\frac{1}{2\Delta t}\right).$$

C-11. $\hat{A}(f)$: Estimate of the value of the frequency response function $A(f)$ is given by

$$\hat{A}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\hat{P}_{yx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)\Big/\hat{P}_{xx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)$$
$$r=0,1,2,\cdots,h.$$

C-12. $\hat{\Phi}(f)$: An estimate of $\Phi(f)$ is given by

$$\hat{\Phi}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\Psi(r)-2\pi\sum_{s=1}^{r}sgn(\Psi(s)-\Psi(s-1))-\pi\frac{r}{h}K$$
$$\Psi(s)=\tan^{-1}\left\{\hat{s}_{yx.K}\left(\frac{s}{h}\frac{1}{2\Delta t}\right)\Big/\hat{c}_{yx.K}\left(\frac{s}{h}\frac{1}{2\Delta t}\right)\right\}$$
$$+\frac{\pi}{2}\left\{1-sgn\left(\hat{c}_{yx.K}\left(\frac{s}{h}\frac{1}{2\Delta t}\right)\right)\right\}sgn\left(\hat{s}_{yx.K}\left(\frac{s}{h}\frac{1}{2\Delta t}\right)\right)$$
$$\text{for }\hat{c}_{yx.K}\left(\frac{s}{h}\frac{1}{2\Delta t}\right)\neq 0$$
$$=\frac{\pi}{2}sgn\left(\hat{s}_{yx.K}\left(\frac{s}{h}\frac{1}{2\Delta t}\right)\right)\quad\text{for }\hat{c}_{yx.K}\left(\frac{s}{h}\frac{1}{2\Delta t}\right)=0$$

where by definition

$$sgn(f)=\begin{cases}1 & (f>0)\\ 0 & (f=0)\\ -1 & (f<0)\end{cases}$$

and

$$\hat{\Phi}(0)=\begin{cases}0 & \text{when } \hat{c}_{yx.K}(0)\geqq 0,\\ \pi & \text{otherwise.}\end{cases}$$

It should be remembered that $\Phi(f)$ and $\hat{\Phi}(f)$ have essential indeterminacy of integral multiples of 2π, and further the sampling fluctuation sometimes introduces into $\hat{\Phi}(f)$ indeterminacy of integral multiples of π.

C-13. $\hat{\gamma}^2(f)$: Estimate of coherency at frequency f is given by

$$\hat{\gamma}^2\left(\frac{r}{h}\frac{1}{2\Delta t}\right)=\left|\hat{A}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)\right|^2\left\{\hat{P}_{yy}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)\Big/\hat{P}_{xx}\left(\frac{r}{h}\frac{1}{2\Delta t}\right)\right\}^{-1}$$
$$r=0,1,2,\cdots,h.$$

C–14. $R(f)$: Relative error $R(f)$ of the present estimate is obtained, though it is a somewhat rough estimate, from the following

$$R(f)=\sqrt{\frac{1}{n-1}\left(\frac{1}{\hat{\gamma}^2(f)}-1\right)F(\delta,\ 2,\ 2(n-1))}$$

$$n=\text{the integer nearest to }\left(\frac{M}{h}\right)\left(2\sum_{n=-k}^{k}a_n^2\right)^{-1},$$

where $F(\delta,\ 2,\ 2(n-1))$ is defined by the relation for $F^2_{2(n-1)}$ following the F-distribution with d.f.s 2 and $2(n-1)$

$$\text{Prob}\ \{F^2_{2(n-1)}\leqq F(\delta,\ 2,\ 2(n-1))\}=\delta\,.$$

Then we have approximately

$$\text{Prob}\ \{\,|\hat{A}(f)-A(f)\,|\leqq R(f)\,|\hat{A}(f)\,|\,\}=\delta\,.$$

In the definition of $R(f)$, $F(\delta,\ 2,\ 2(n-1))$ should be replaced by $F(\delta,\ 1,\ (n-1))$ at $f=0$ and $1/2\Delta t$. We also have

$$\text{Prob}\ \{||\hat{A}(f)\,|-|A(f)\,||\leqq R(f)\,|\hat{A}(f)\,|\ \text{and}$$
$$|\arg\hat{A}(f)-\Phi(f)\,|\leqq\sin^{-1}R(f)\}\geqq\delta\,.$$

$R(f)$ should be put equal to 1.00 to indicate the relative error greater than 100%, when the value inside the square root of the definition of $R(f)$ is greater than 1 or less than 0.

C–15. $\{a_n\}$: $\{a_n\}$'s are given by

	a_0	$a_1=a_{-1}$	$a_2=a_{-2}$	$a_3=a_{-3}$
$W_1=($	0.5132	0.2434	0	0 $)$
$W_2=($	0.6398	0.2401	−0.0600	0 $)$
$W_3=($	0.7029	0.2228	−0.0891	0.0149).

When there is a significant change, say, of order greater than 10%, between the results, it is advisable to repeat the whole computing process using $2h$ in place of h and at the same time with correspondingly increased M, if necessary. Usually the peaks tend to be higher and troughs to be deeper in the order of W_1, W_2 and W_3.

Comments

M–1. The following relation should be taken into account in the stage of planning the experiment:

$$\frac{\sqrt{E|\hat{A}(f)-A(f)|^2}}{|A(f)|} \doteqdot \sqrt{\frac{h}{M}\left(\frac{1}{\gamma^2(f)}-1\right)},$$

where $\gamma^2(f)$ is the value of coherency at frequency f and is defined for stationary $x(t)$ and $y(t)$, with power spectral densities $P_x(f)$ and $P_y(f)$ respectively ;

$$\gamma^2(f)=\frac{|A(f)|^2 P_x(f)}{P_y(f)} .$$

Thus if the frequency ranges where $\gamma^2(f)\geqq 1/2$ holds are of the experimenter's main concern it will be advisable to take h satisfying

$$h\Delta t \geqq 10 M\Delta t$$

to get a fairly reasonable result, but if the values in the range where $\gamma^2(f)\geqq 3/4$ are of his main concern,

$$h\Delta t \geqq 5 M\Delta t$$

may replace the above inequality.

M-2. When the discrepancy between the results obtained by applying the windows W_1, W_2 and W_3 at some frequency is significant, it can be considered to suggest that there is something which causes difficulty in the estimation procedure. As to the design principle of W_i's confer the paper by the present author [2].

M-3. In selection of h, the shapes of $\hat{P}_{xx}(f)$, $\hat{c}_{yx.K}(f)$ and $\hat{s}_{yx.K}(f)$ should be taken into account. The difference between the adjacent values, given at $f=(r/2h\Delta t)$s, should be kept less than half the larger one. It is also desirable to keep the overall variation of the level of these values in the range of 1/100 or at least 1/1000 of the corresponding maximum values, respectively. The very sharp peak of $\hat{P}_{yy}(f)$ may cause some trouble in the calculation of $\hat{\gamma}^2(f)$, but for the estimation of $A(f)$ it indicates only the increase of sampling variability. Careful inspection of the auto- and cross-correlograms greatly facilitates the decision of the necessary constants for the computation.

M-4. Proper use of the prewhitening operation reduces greatly the bias otherwise introduced by the existence of undesirable frequency components. Digital filtration as well as analog filtration can be used quite effectively to reduce the distortion during the numerical processing. When the filters of one and the same characteristic are applied both to input $x(t)$ and output $y(t)$, no correction afterwards is necessary for the estimation of $A(f)$. Unnecessary high frequency components should be

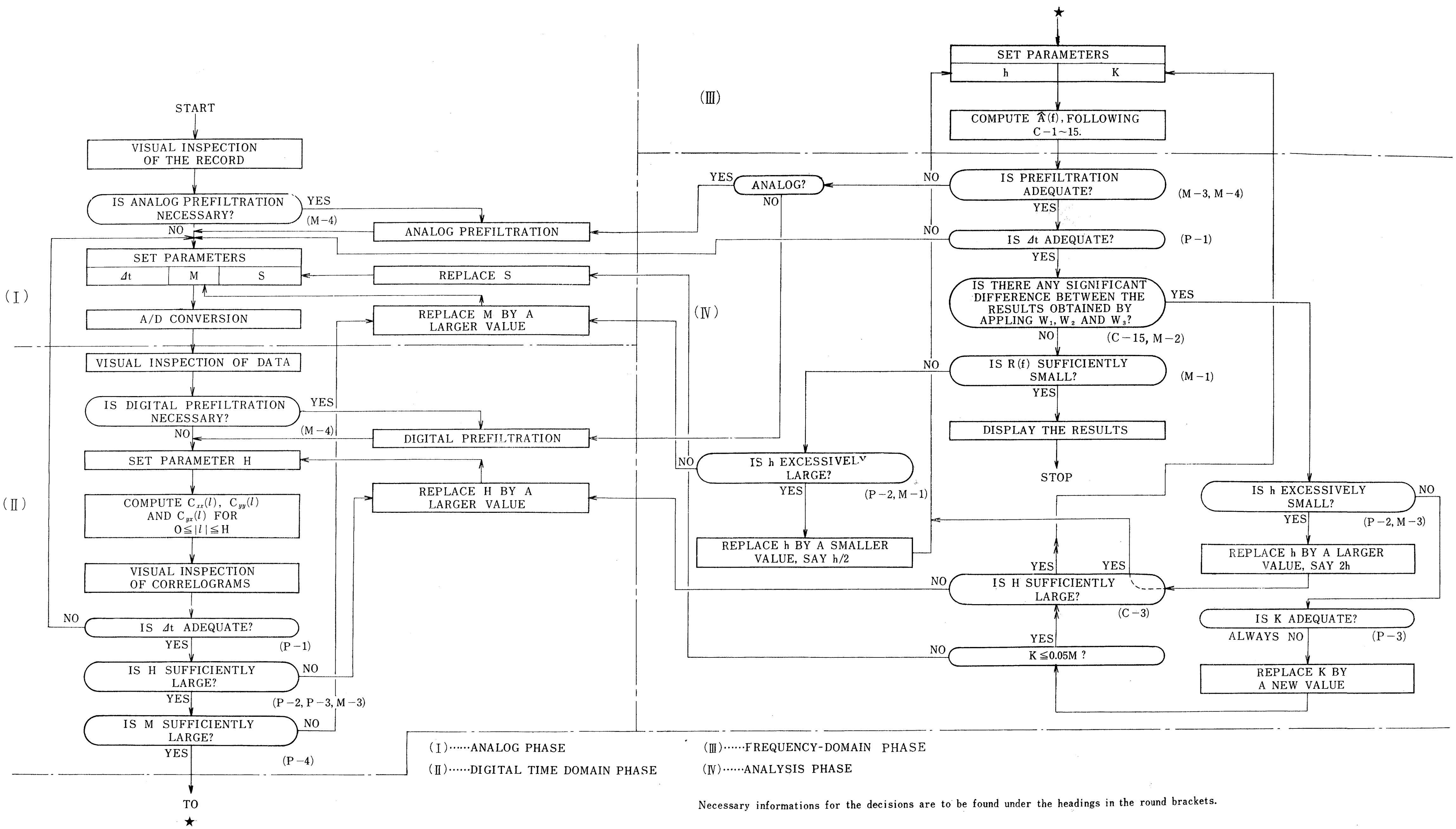

Fig. 2. Flow diagram of the estimation procedure

eliminated by the analog filtration to allow a large value of Δt for sampling.

M-5. In the calculation of $R(f)$, the value

$$\delta = 0.95$$

was used throughout our present research project, and the results were considered to give fairly reasonable indication of the sampling error. When $R(f)$ showed the value 1.00, the estimate of $\Phi(f)$ often showed sudden changes of magnitude $\pm\pi$, which shows unreliability of the result. For this point confer the results reported in other papers of this supplement. The following approximation formulae for F-values can be conveniently used for computer application.

$$F(2,\ n,\ 0.95) \fallingdotseq 3.00 + \frac{10.00}{n-1.40}\ ,$$

$$F(1,\ n,\ 0.95) \fallingdotseq 3.84 + \frac{10.00}{n-1.40}\ .$$

M-6. The input $x(t)$ need not be a stationary process, but for practical applications of the method the systems under consideration usually contains non-linearity and the present method gives the frequency characteristics of the corresponding linearized system. Thus the result has a technical meaning only under the same type of input $x(t)$. This fact should not be overlooked, and the application of the method to records of the system under various types of input $x(t)$ of practical importance is especially necessary to get a valuable conclusion. For this point see the paper by I. Kaneshige [pp. 49–57 of this supplement] and that by I. Kaneshige and the present author [pp. 99–102 of this supplement].

M-7. The above stated computing scheme is intended only for pilot estimation, and the final technical conclusions should be based on the results of repeated applications of the method. Constants for the computation should be modified intentionally to check the reliability and validity of the results.

Flow diagram of the estimation procedure

The estimation procedure which was adopted throughout our research project is flow-diagrammed in Fig. 2.

4. Some conclusions from practical applications of the method

In this section we shall describe some conclusions obtained from practical applications of our estimation procedure.

4–1. The method can be applied quite easily, and the flow diagram illustrated in Fig. 2 of section 3 will clarify that to take the method using cross-spectra as something sophisticated is merely a prejudice. Necessary decisions for the analysis are between yes and no to the inquiries in the round blocks in Fig. 2. Without answering these questions explicitly or implicitly, anyone can scarcely arrive at a reliable valid conclusion of the actual behavior of the system under consideration, even if he adopts any other method of estimation. It is a significant point of the present method that these inquiries of technical importance are stated explicitly. Thus, at present, the method of estimation in the frequency-domain, described in this paper, seems to be better suited for the ordinary analysis of the response characteristics of a system than the method in the time-domain. By the latter, the impulse response of a system is directly calculated, but the input to a system sometimes is of a narrow band-width so that it makes the notion of impulse response merely a mathematical fiction. For this point also confer the conclusion of I. Nakamura's paper [pp. 41–47 of this supplement].

4–2. As is commonly recognized, the power spectral analysis is a powerful fundamental method for the analysis of stationary random phenomena. It clarifies what are hidden in apparently random fluctuations of the object; however, it does not explain where they are from. The present analysis of cross-spectra clarifies which of the two records $x(t)$ and $y(t)$ precedes in time and, if properly used, may trace the flow of random signals. This is the point which the present method adds to the hitherto used power spectral analysis, and will greatly contribute to the improvement of measurement and analysis techniques in many research fields. For this last statement, confer the paper by H. Sato [pp. 71–78 of this supplement].

In some of the applications, $\hat{A}(f)$ obtained by the present method may not directly mean any frequency response function of a physical system and merely describes a relation between $x(t)$ and $y(t)$. Even such a relation can afford an important information of the object under investigation. An example of application of this sort will be found in K. Suhara's paper [pp. 89–98 of this supplement].

4–3. The present method of estimation of the frequency response function persuits the relation between input and output. Thus it can find that a stable relationship may hold even if the spectra of the input and output are greatly changed in two experiments. Here the main point of concern is the stationality of the relationship and not the stationality of the whole process. The results reported in this supplement by K. Suhara [pp. 89–98] and I. Kaneshige [pp. 49–57] will clarify this point.

4–4. As is obvious, the signal to noise ratio in the original record cannot be improved by the use of a digital filter after sampling, and proper use of an analog filter before the analog to digital conversion is most profitable. Nevertheless, the digital filter can be used successfully to reduce the distortion, due to the leakage through side robes of the spectral window, during the numerical computation. For the analyses of data by S. Takeda [pp. 59–63 of this supplement] and by T. Kinosita [pp. 79–87 of this supplement] digital filters were applied.

The most difficult input for an adequate treatment will be the one which contains a periodic component with significant higher harmonics.

4–5. Successive applications of the windows W_1, W_2 and W_3 greatly facilitate to get necessary informations as to the decision whether the result is satisfactory or not. Comparison of the three results obtained prevents one from an erroneous hasty conclusion.

4–6. In case the assumption of local flatness of the power spectrum of the input is not admitted, use of W_1 in the final display of the result is recommended.

4–7. The relative error $R(f)$ for $\delta=0.95$ seems to be quite useful as an indicator of the magnitude of sampling fluctuation, and its value 1.00 (or 100% error) warns the analyst of the spurious high amplitude gain and the entirely unreliable phase shift.

4–8. The present method treats only one aspect of a phenomenon at a time, and rather low coherencies in some applications suggest that there may be significant contributions from inputs ignored in the analyses. This means that the further improvement of the method must be made to accept the multi-dimensional input cases.

5. Data gathering and processing system

All the numerical results, except one of I. Nakamura, were obtained by the TSK–III computing system, with central processor HIPAC 103, of our institute.* As to the original records, they were given in digital form by Y. Yamanouchi and T. Kinosita, in graphic form by H. Nakamura and in voltage magnetic recording (PWM) form by the others. The graphically represented time histories of the fluctuations of the load of a generator and of the water level of the surge tank were converted automatically into analog voltage fluctuations by using the oscillogram tracer OT–1 which was recently developed by the Ōyō Denki Kenkyusyo

* All the necessary programmings were performed by the present author and Mrs. Y. Sinozaki. The data of I. Nakamura were processed on OKITAC 5090 computer by courtesy of Dr. M. Sugawara of our institute and the programs were prepared by Miss Y. Katuyama.

(Applied Electrics Laboratory) and prepared for the present research project.

Analog voltages in the range of ± 20 volts were converted by A/D converter into digital form of 11 bits with sign in 160 microseconds. This time length for conversion introduces the so-called aperture effect or the indeterminacy of the sampling instant, but its contribution to a pure sinusoidal wave is at most 5% in amplitude at 50 c.p.s. and is proportional to the frequency of the input. Taking into account that the analog voltages treated in our project had their significant powers below 20 c.p.s., the contribution of the aperture effect can be considered to be negligibly small. Analysis of the frequency characteristics of this noise will be an interesting problem.

The HIPAC 103 is a 48 bits computer, using parametron, with core-memory of 4096 words and addressable drum-memory of 4096 words. The addition and multiplication operations are performed in 0.4 millisecond and 1.8 millisecond, respectively, and thus taking about 45 minutes for the computation of correlograms of phase II of Fig. 2 for $M=1500$ and $H=100$ and about 5 minutes for the Fourier analysis of phase III of Fig. 2 for $h=50$. Besides, the necessary printings, C_{xx}, C_{yy}, C_{yx}, $\bar{P}_{xx}$, $\bar{P}_{yy}$, $\bar{c}_{yx}$ and $\bar{s}_{yx}$ are punched out on paper tapes. The resultant $\hat{P}_{xx}$, $\hat{P}_{yy}$, $|\hat{A}(f)|$, $\hat{\Phi}(f)$, $\hat{\gamma}^2(f)$ and $R(f)$ are displayed in minutes in a graphical form by using a CALCOMP digital plotter.

These figures and the numerical results reported in other papers of this supplement show that, by using the microsecond speed computer, the present estimation procedure can be adopted as an economical on-line method of measurement for many industrial applications. Our system is schematically illustrated in Fig. 3. The system will soon be modified to a complete hybrid system by adding an analog computer with linkage, and the operations in analog phase I of Fig. 2 will greatly be facilitated.

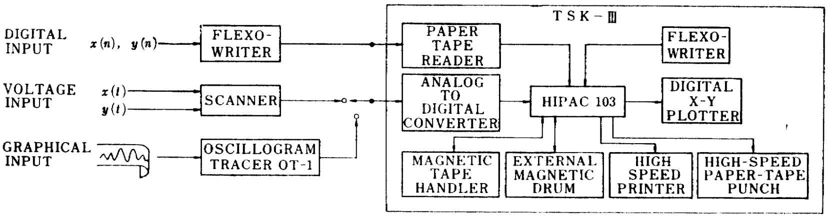

Fig. 3 Data gathering and processing system

Acknowledgment

The author expresses his hearty thanks to the participants of the project for kind discussions of the method and to Mr. S. Nakaya, the former chief of the general affairs division of our institute, for prompting the project.

Thanks are also due to the members of the computing room for the computations on TSK-III, and to Mrs. Y. Sinozaki for her help in programming of the computer.

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[2] H. Akaike, "On the design of lag window for the estimation of spectra," *Ann. Inst. Stat. Math.*, Vol. 14 (1962), pp. 1-21.

Corrections to "On classification by the statistics R and Z" by S. John, this Annals, 14 (1963), 237–246

1) θ^{-1} in expression (5.2) should be read as $(-\theta)^{-1}$.

2) The summation with respect to s in expression (5.12) should be read from 0 to r in the first term and from $r+1$ to ∞ in the second term.

Corrections to the Supplement III (1964) of this Annals

page 6, line 25, "sample" should be deleted.

page 10, line 8, $sgn(\Psi(s)-\Psi(s-1))$ should be multiplied by a factor

$$\tfrac{1}{2}\{sgn(|\Psi(s)-\Psi(s-1)|-\pi)+1\}sgn(||\Psi(s)-\Psi(s-1)|-\pi|)$$

and $+\hat{\Phi}(0)$ should be added at the end of the line.

page 12, lines 8 and 11, M and h should be interchanged.

page 12, line 21, "of $\hat{P}_{xx}(f)$ and $|\hat{P}_{yx.K}(f)|^2$," should be added to "$\cdots$ level of these values" at the end of the line.