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TESTING WHETHER SURVIVAL FUNCTION IS HARMONIC NEW BETTER THAN USED IN EXPECTATION

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Summary

Statistical procedures to test that a life distribution is exponential against the alternative that it is harmonic new better than used in expectation (HNBUE) are considered.

1. Introduction

In performing reliability analyses, it has been found very useful to classify life distributions using the concepts of aging (wearout). The most well known classes of life distributions (i.e. distribution function with $F(0-)=0$) based on some aging property are: 1) the increasing failure rate (IFR) class; 2) the increasing failure rate in average (IFRA) class; 3) the new better than used (NBU) class; 4) the decreasing mean residual life (DMRL) class; and 5) the new better than used in expectation (NBUE) class. Each of these classes has a dual class. These are named DFR, DFRA, NWU, IMRL and NWUE, respectively.

Rolski [26] introduced a new class of life distributions called harmonic new better than used in expectation (HNBUE) with dual HNWUE. Properties of such life distributions have been treated in Klefsjö [16], [17].

DEFINITION 1.1. A life distribution F is HNBUE if

$$\int_t^\infty \bar{F}(x)dx \leqq \mu \exp(-t/\mu) \qquad \text{for all } t \geqq 0, \tag{1.1}$$

where $\bar{F}=1-F$, $\frac{1}{\lambda}=\mu=\int_0^\infty \bar{F}(x)dx<\infty$. If the reversed inequality holds in (1.1) then F is HNWUE.

By using the mean residual life

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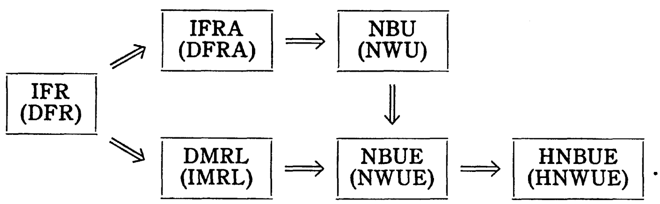
$$\mu_F(x)=\begin{cases} \dfrac{\int_x^\infty \bar{F}(y)dy}{\bar{F}(x)} & \text{if } \bar{F}(x)>0 \\ 0 & \text{if } \bar{F}(x)=0 \end{cases}$$

it can be proved that the condition (1.1) is equivalent to

$$\text{(1.2)} \qquad \frac{1}{\frac{1}{t}\int_0^t \mu_F^{-1}(x)dx} \leqq (\geqq) \mu \qquad \text{for all } t \geqq 0 \, .$$

(cf. Klefsjö [16]), where $\mu_F^{-1}(x)$ refers to the reciprocal of $\mu_F(x)$. The condition (1.1) means that the integral harmonic value of the mean residual life of a unit at age x is less (greater) than or equal to the integral harmonic mean value of a new unit. This is the reason why Rolski [26] used the name HNBUE (HNWUE).

The following chain of implication exists among the six classes of distributions (cf. Basu, Ebrahimi and Klefsjö [7])



Basu and Ebrahimi [6] explored the possibility of extending the HNBUE (HNWUE) class by defining the k-order HNBUE (k-order HNWUE) class of distributions where HNBUE (HNWUE) corresponds to the case when $k=1$. It was shown that though $\boxed{\text{HNBUE}} \Longrightarrow \boxed{k\text{-HNBUE}}$, unfortunately $\boxed{k\text{-HNWUE}} \Longrightarrow \boxed{\text{HNWUE}}$. Thus HNBUE (HNWUE) seems to be a more natural class of distributions.

The boundary members of the HNBUE class, obtained by insisting on equality in (1.2), are of course exponential distributions. In this paper we consider the inferential problem of testing

$$H_0 : F(x)=1-\exp(-\lambda x) \, , \qquad x \geqq 0, \ \lambda > 0$$

versus

$$\text{(1.3)} \qquad H_1 : F \text{ is HNBUE (and not exponential)} \, ,$$

or

$$H_2 : F \text{ is HNWUE (and not exponential)} \, ,$$

on the basis of a random sample $T_1, \cdots, T_n$ from the distribution F. We assume that F is absolutely continuous.

During recent years some tests have been suggested for testing $H_0: F$ is exponential versus $H_1: F$ is ∇ but not exponential, where ∇ denotes IFR, IFRA, NBU, NBUE or DMRL. Such tests were proposed e.g. by Proschan and Pyke [25], Barlow [1], Bickel and Doksum [10], Klefsjö [18], and Deshpande [13] when ∇ is IFR, by Barlow and Campo [3], Bergman [9], and Klefsjö [18] when ∇ is IFRA, by Hollander and Proschan [14], Koul [19], [20], Kumazawa [22], and Deshpande and Kochar [12] when ∇ is NBU, by Hollander and Proschan [15], Koul [20] and Koul and Susarla [21] when ∇ is NBUE and by Hollander and Proschan [15] and Klefsjö [18] when ∇ is DMRL.

Since NBUE implies HNBUE, that is, HNBUE is the largest available class of distributions with aging property, therefore the test of H_0 versus H_1 that we propose focuses on a larger class of alternative distributions. For application along with some other important properties of HNBUE see Klefsjö [16], [17].

The scaled total time on test (TTT-transform) was introduced by Barlow and Campo [3] and discussed in more details by Barlow [5]. This concept has proven to be very useful in the statistical analysis of life data. In Section 2 of this paper, we have given two test statistics for testing H_0 against H_1. Both test statistics are in terms of empirical TTT-transform. The asymptotic behaviour and the consistency of test statistics are discussed in Sections 3 and 4. Finally in Section 5, the techniques of Section 2 are illustrated using two examples.

2. Test statistics based on the empirical scaled TTT-transform

In this section, we will obtain two test statistics for testing (1.3) based on the empirical scaled TTT-transform.

We start with the following definitions.

DEFINITION 2.1. Let F be a life distribution with finite mean μ. The *TTT-transform* of F is then defined by

$$H_F^{-1}(t)=\int_0^{F^{-1}(t)} \bar{F}(s)ds\,, \qquad \text{for } 0\leqq t\leqq 1 \tag{2.1}$$

where $F^{-1}(t)$ is the inverse function of F.

DEFINITION 2.2. Let F be a life distribution with finite mean μ. The scaled TTT-transform of F is then defined by

$$\phi_F(t)=\frac{H_F^{-1}(t)}{H_F^{-1}(1)} \qquad \text{for } 0\leqq t\leqq 1\,, \tag{2.2}$$

where by (2.1), $H_F^{-1}(1)=\mu$.

Remark 1. If F is exponential then the scaled TTT-transform is given by $\phi_F(t)=t,\ 0\leqq t\leqq 1$.

Assume that $t(1)\leqq t(2)\leqq\cdots\leqq t(n)$ is an ordered sample from a life distribution F (and let $t(0)=0$). Further let

$$(2.3)\qquad Y_j=\sum_{k=1}^{j}(n-k+1)(t(k)-t(k-1))\ ,\qquad j=1,\cdots,n$$

denote the total time on test at $t(j)$. A natural choice of estimator of the scaled TTT-transform is the empirical scaled TTT-transform

$$(2.4)\qquad \phi_n(t)=\frac{H_n^{-1}(t)}{H_n^{-1}(1)}\ ,\qquad 0\leqq t\leqq 1\ ,$$

where $H_n^{-1}(t)=\int_0^{F_n^{-1}(t)}\bar{F}_n(s)ds$ for $0\leqq t\leqq 1$, F_n is the empirical distribution function and $\bar{F}_n=1-F_n$. Calculations show that

$$(2.5)\qquad \phi_n\left(\frac{j}{n}\right)=S_j\ ,\qquad j=0,1,\cdots,n$$

where

$$S_j=\frac{Y_j}{Y_n}\ ,\ j=0,1,\cdots,n,\ Y_0=0,\ \text{and}\ Y_n=\sum_{k=1}^{n}t(k)\ .$$

One way to get a test statistic for testing exponentiality against HNBUE is to use (1.2), (2.2) and the fact that F is HNBUE (HNWUE) if and only if

$$(2.6)\qquad \int_0^t\left\{\frac{1-F(x)}{1-\phi_F(F(x))}-1\right\}dx\geqq(\leqq)0\ .$$

The left hand side of the HNBUE inequality (2.6) has been estimated for the specified times $t(1),\cdots,t(n)$, and summed up, that is, the estimate A_n is given by

$$(2.7)\qquad A_n=\sum_{r=1}^{n}\sum_{j=1}^{r}\left[\frac{(n-j+1)/n}{1-S_{j-1}}\right][t(j)-t(j-1)]$$
$$=\frac{1}{n}\sum_{j=1}^{n}\sum_{r=j}^{n}\frac{D_j}{1-S_{j-1}}-Y_n=\left[\frac{1}{n}\sum_{j=1}^{n}(n-j+1)\frac{D_j}{1-S_{j-1}}\right]-Y_n\ ,$$

where $D_j=(n-j+1)(t(j)-t(j-1))$, $j=1,\cdots,n$ and Y_n and S_j are given by (2.3) and (2.5) respectively. The subsequent test criterion using A_n then, consist of noting whether the estimated inequality (2.6) is satisfied significantly on the average over times $t(1),\cdots,t(n)$, that is, we expect a positive (negative) value of A_n if F is HNBUE (HNWUE), but not exponential.

Our second test statistic for testing H_0 against H_1 is based on the fact that F is HNBUE (HNWUE) if and only if

$$\ln(1-\psi(F(t))\leqq(\geqq)-\frac{t}{\mu}\ , \qquad \text{for all } t\geqq 0\ . \tag{2.8}$$

The left hand side of the HNBUE inequality (2.8) has been estimated for the specified times $t(1),\cdots,t(n)$, and summed up, with $\sum_{j=1}^{n}\frac{t(j)}{\mu}$ being estimated by n, that is, estimate B_n is given by

$$B_n=\left[\sum_{j=1}^{n}\ln(1-S_{j-1})\right]+n\ . \tag{2.9}$$

The subsequent test criterion using B_n, then, consist of noting whether the estimated inequality (2.8) is satisfied significantly on the average over times $t(1),\cdots,t(n)$.

We have not been able to find the distribution of A_n under H_0 (in closed form) for different n. It is shown the complexity of the distribution of A_n increases very rapidly as the number of observations increases.

The following theorem gives the distribution of A_n under H_0 for $n=2$.

THEOREM 1. *Under the null hypothesis*

$$f_{A_2}(t)=\lambda\exp(-2\lambda|t|)\ , \qquad -\infty<t<\infty\ .$$

PROOF. From (2.7),

$$A_2=2t(1)-Y_2/2=\sum_{i=1}^{2}a_i t(i)\ , \tag{2.10}$$

where $a_1=3/2$, $a_2=-1/2$. Using the fact that $2t(1)$ and $t(2)-t(1)$ are independent under H_0 and have the same exponential distribution with parameter λ (cf. Barlow and Proschan [4], p. 59) we get

$$f_{A_2}(t)=\begin{cases}\lambda\exp(2\lambda t)\ , & -\infty<t<0\\ \lambda\exp(-2\lambda t)\ , & 0<t<\infty\ .\end{cases}$$

Remark 2. For an arbitrary $n>2$, we can write

$$A_n=nt(1)+\frac{Y_n}{n}\left[\left(\sum_{j=2}^{n-1}(n-j+1)\right)-n-1\right]$$
$$-\frac{Y_n}{n}\sum_{j=2}^{n-1}(n-j+1)\frac{D_{j+1}+\cdots+D_n}{D_j+\cdots+D_n}\ .$$

Since $\frac{D_{j+1}+\cdots+D_n}{D_j+\cdots+D_n}$ has Beta distribution with parameters $(n-j, 1)$ and $\frac{D_{j+1}+\cdots+D_n}{D_j+\cdots+D_n}$'s are independent, $j=2,\cdots,n-1$, the distribution of $\sum_{j=2}^{n-1}\frac{D_{j+1}+\cdots+D_n}{D_j+\cdots+D_n}$ is the convolution of $(n-2)$ Beta distributions. It is difficult even to write the distribution of $\sum_{j=2}^{n-1}\frac{D_{j+1}+\cdots+D_n}{D_j+\cdots+D_n}(n-j+1)$ in closed form.

The following theorem gives the distribution of B_n under H_0.

THEOREM 2. *Under the null hypothesis*

$$(2.11)\qquad f_{B_n}(t)=\begin{cases}\dfrac{1}{(n-2)!}(n-t)^{n-2}\exp(t-n) & \textit{for } -\infty<t<0\\ 0 & \textit{otherwise}.\end{cases}$$

PROOF. Using the fact that $\sum_{j=1}^{n}\ln(1-S_{j-1})$ has the same distribution as $\sum_{j=1}^{n-1}\ln X_j$ where X_j's are independent and have uniform distribution over $[0, 1]$ (cf. David [11], p. 99) we get

$$(2.12)\qquad f_{W_n}(t)=\begin{cases}\dfrac{1}{(n-2)!}(-t)^{n-2}\exp(t) & \text{for } -\infty<t<0\\ 0 & \text{otherwise},\end{cases}$$

where $W_n=\sum_{j=1}^{n}\ln(1-S_{j-1})$ and hence (2.11).

Table 1. Critical values of the test statistic B_n with level α

n \ α	Lower tail .005	.01	.05	.1	Upper tail .005	.01	.05	.1
3	−4.5	−3.5	−1.75	−.8	2.9	2.8	2.65	2.4
4	−5.14	−4.23	−2.4	−1.2	3.5	3.15	3.05	2.95
5	−6.24	−4.4	−2.68	−1.8	4.75	4.2	4.1	3.9
6	−7.07	−5.17	−3.5	−2.25	5.7	5.4	5.2	5.03
7	−7.28	−6.76	−4.16	−2.9	6.3	6.1	5.8	5.45
8	−7.52	−6.86	−3.6	−3.1	6.6	6.5	6.4	6.15
9	−8.85	−7.19	−4.25	−3.2	6.8	6.55	6.45	6.3
10	−9.15	−7.5	−4.45	−3.4	6.95	6.8	6.6	6.5
11	−9.55	−8.52	−4.9	−3.8	7.12	6.9	6.78	6.7
12	−9.8	−8.75	−5.3	−4.3	7.5	7.3	7.15	7.1
13	−10.14	−8.9	−5.75	−4.9	8.6	8.35	8.12	7.95
14	−11.4	−9.2	−6.17	−5.2	9.7	9.4	9.3	9.15

Using the equation (2.11) we give critical values for B_n for $n=3(1)14$.

3. The asymptotic behavior of A_n and B_n

In this section we study the behaviour of our test statistics. First we prove the following lemma.

LEMMA 1. *Under the null hypothesis, $A_n/\sqrt{n}$ has asymptotically the same distribution as* $Y_n/n\left(\frac{1}{\sqrt{n}}\sum_{j=2}^{n-1}\left((n-j)-(n-j+1)\frac{D_{j+1}+\cdots+D_n}{D_j+\cdots+D_n}\right)\right)$.

PROOF. From (2.7), we can write

$$A_n/\sqrt{n}=\frac{Y_n}{n}\left[\left(\sum_{j=2}^{n-1}(n-j)-(n-j+1)\frac{D_{j+1}+\cdots+D_n}{D_j+\cdots+D_n}\right)\Big/\sqrt{n}\right]+\frac{n(t(1))}{\sqrt{n}}-\frac{Y_n}{n\sqrt{n}}\ .$$

Since under null hypothesis, $n(t(1))$ has exponential distribution and Y_n/n goes to $1/\lambda$ in probability, therefore $n(t(1))/\sqrt{n}$ and $Y_n/n\sqrt{n}$ both go to zero in probability.

Remark 3. We should mention that $t(1)$ and Y_n/n are asymptotically independent (cf. David [11], p. 270).

THEOREM 3. *Under the null hypothesis,*

$$\lim_{n\to\infty} \mathrm{P}\left(\frac{\lambda A_n}{\sqrt{n}}\leqq t\right)=\Phi(t)\ , \tag{3.1}$$

where $\Phi(t)$ is the standard normal distribution.

PROOF. Use Lemma 1 and Lindeberg's condition.

The following theorem gives the asymptotic distribution of B_n.

THEOREM 4. *Under the null hypothesis*

$$\lim_{n\to\infty} \mathrm{P}\left(\frac{B_n}{\sqrt{n}}\leqq t\right)=\Phi(t)\ . \tag{3.2}$$

PROOF. Since $\sum_{j=1}^{n}\ln(1-S_{j-1})$ has the same distribution as $\sum_{j=1}^{n-1}\ln X_j$ where X_j's are independent and have the same uniform distribution over $[0,1]$, by central limit theorem $\sum_{j=1}^{n-1}\log X_j$ has asymptotically normal distribution with mean $-(n-1)$ and variance $n-1$. The equation (2.9) can be written as

$$B_n=\left[\sum_{j=1}^{n}\ln(1-S_j)+(n-1)\right]+1 .$$

From this (3.2) follows.

Remark 4. The distribution of B_n is invariant under λ but the distribution of A_n depends on λ. However, by taking a consistent estimator $\hat{\lambda}_n=n/Y_n$, one can consider a test based on $\hat{\lambda}_n A_n$ whose asymptotic distribution is independent of the λ. We should mention that if λ is known one can use A_n.

Remark 5. Since $\psi_n(x)$ is a consistent estimator of $\psi_F(x)$ (cf. Barlow [5]), therefore $\int_0^t\left\{\frac{1-F_n(x)}{1-\psi_n(x)}-1\right\}dx$ and $\ln\{(1-\psi_n(F(t))\}+\frac{nt}{Y_n}$ are consistent estimators of $\int_0^t\left\{\frac{1-F(x)}{1-\psi(x)}-1\right\}dx$ and $\ln\{(1-\psi(F(t))\}+\frac{t}{\mu}$ for all t.

Remark 6. While the first draft of this paper was being written one related work was brought to our attention. Borges, Proschan and Rodrigues [8] have used sample coefficient of variation as a test for exponentiality versus NBUE. We think that one can use the same test statistic to test exponentiality versus HNBUE.

Since it is difficult to find the distributions of our test statistics under alternative hypothesis (even asymptotic distributions), we have used Monte Carlo approach to study the power of our statistics. In our study we have also included two statistics A_2 and B, studied by Klefsjö [18], which are in the following form

$$A_2=\sum_{j=1}^{n}\frac{\alpha_j D_j}{S_n} \tag{3.3}$$

and

$$B=\sum_{j=1}^{n}\left(\frac{\beta_j D_j}{S_n}\right), \tag{3.4}$$

where $\alpha_j=\frac{1}{6}\{(n+1)^3 j-3(n+1)^2 j^2+2(n+1)j^3\}$, $\beta_j=\frac{1}{6}\{2j^3-3j^2+j(1-3n-3n^2)+2n+3n^2+n^3\}$. We have simulated the power of tests with significance level $\alpha=.05$ for some Weibull and gamma alternatives for $n=20$. The power estimates are based on 2000 simulations each. The results are given in Table 2.

From Table 2 we observe that B_n has largest power values. We think the main reason for that is B_n tests for exponential distribution against a larger class of distributions.

Table 2. Power estimates based on 2000 samples of size $n=20$ with $\alpha=.05$.

	A_n	B_n	A_2	B
$\bar{F}(x)=\exp(-x^{1.5})$	.62	.68	.41	.59
$\bar{F}(x)=\exp(-x^{2})$	.83	.97	.78	.85
$\bar{F}(x)=\exp(-x^{.8})$	.54	.59	.12	.43
$\bar{F}(x)=\int_x^\infty te^{-t}dt$	.68	.75	.28	.72

4. Efficiency of the tests

Using the result from Barlow et al. [2], pp. 284–285, we get that the test statistic B_n is unbiased and consistent.

To compute the Pitman efficiency of test statistic B_n (for a description of Pitman efficiency see Lehmann [23], pp. 371–380) we need the following

THEOREM 5. *If $\{F_n\}$ is a sequence of alternative distributions contiguous to $G_1(x)=1-\exp(-x)$ (for a definition of contiguity see Barlow et al. [2]). Then*

$$\sqrt{n}\left(\frac{-B_n}{n}-\mu(F_n)\right) \tag{4.1}$$

converges in distribution to a standard normal distribution, where

$$\mu(F_n)=\left[\int_0^\infty xdF_n(x)-\int_0^\infty x(1-F_n(x))dx-1\right]\left[\int_0^1 (F_n(u))du\right]^{-1}+1\,. \tag{4.2}$$

PROOF. Use theorem (6.11) in Barlow et al. [2].

When testing a simple hypothesis $\theta=\theta_0$ against an alternative hypothesis $\theta>\theta_0$ (say) using a test statistic Υ which is asymptotically normally distributed with mean $\mu(\theta)$ and variance $\sigma^2(\theta)/n$ the efficacy for calculating Pitman efficiency is given by $E_{F_n}(\Upsilon)=(\mu'(\theta_0))^2/\sigma^2(\theta_0)$, where the dash indicates differentiation. In our case this means that $E_{F_n}(B_n/n)=(\mu'(F_n))^2/\theta=\theta_0$, where θ_0 corresponds to the exponential distribution.

We calculate $E_{F_n}(B_n/n)$ for linear failure rate, Makeham, Pareto, Weibull and gamma alternatives given respectively by

$$F_1(x)=1-\exp\left(-\left(x+\frac{1}{2}\theta x^2\right)\right) \qquad \text{for } \theta\geqq 0,\ x\geqq 0$$

$$F_2(x)=1-\exp(-(x+\theta(x+e^{-x}-1))) \qquad \text{for } \theta\geqq 0,\ x\geqq 0$$

$$(4.3) \qquad F_3(x)=1-(1+\theta x)^{-1/\theta} \qquad \text{for } \theta\geqq 0,\ x\geqq 0$$

$$F_4(x)=1-\exp(-x^{\theta}) \qquad \text{for } \theta>0,\ x\geqq 0$$

$$F_5(x)=\frac{1}{\Gamma(\theta)}\int_0^x t^{\theta-1}e^{-t}dt \qquad \text{for } \theta>0,\ x\geqq 0\,.$$

For F_1, F_2 and F_3, $\theta_0=0$, and for F_4 and F_5 H_0 corresponds to $\theta=\theta_0=1$. Calculations give the following efficacy values

	LFR	Makeham	Pareto	Weibull	Gamma
$E_{F_i}(B_n/n)$	1.000	.063	1.000	1.000	.250

As an example, for the LFR case, the efficiency of the statistic B_n with respect to the best test proposed by Kumazawa [22], is 1.28. Pitman ARE of B_n compared to other tests can be similarly computed.

5. Examples

The techniques of Sections 2 and 3 will be illustrated by means of the following examples.

Example 1. Consider the data given in Proschan [24]. The data represent the failures of airconditioning equipment in 13 different aircrafts. We want to test if failure times for each plane follow exponential distribution or if they follow HNBUE (HNWUE) distribution. The computations associated with B_n and A_n are given in Tables 3 and 4 respectively.

Let $H_1(H_2)$ denote the alternate hypothesis that F is HNBUE (HNWUE) and not exponential. Table 3 summarizes the results of the tests for each plane with significance level $\alpha=.05$ when B_n is used. For plane number 7909, 7911, 7913 and 8045 the alternative hypothesis was H_1 and for the remaining planes the alternative hypothesis considered was H_2. In all cases H_0 was accepted. For small sample size we have used Table 1.

Table 4 summarizes the results obtained when A_n is used. In this case the test is available only for $n=2$ and for large samples. The conclusions reached are similar to that in Table 3.

Barlow et al. [2] tested $H_0: F$ is exponential distribution against $H: F$ is DFR (but not exponential) for planes 7907, 7908, 7915, 7916 and 8044. There is a good agreement between the results obtained by Barlow et al. [2] and our results. Also, there is good agreement between our results and results obtained by Proschan [24].

Table 3.

Plane	Sample size	Observed value of statistic B_n	Conclusions ($\alpha=.05$)
7907	6	$B_6=.81$	2**
7908	23	$B_{23}=6.86$ ($Z=1.3$)*	2**
7909	29	$B_{29}=-.7944$ ($Z=-.14$)*	1*
7910	15	$B_{15}=5.76$ ($Z=1.45$)*	2**
7911	14	$B_{14}=-2.11$	1*
7912	30	$B_{30}=+7.01$ ($Z=+1.2$)*	2**
7913	27	$B_{27}=-4.08$ ($Z=-.78$)*	1*
7914	24	$B_{24}=.15$ ($Z=.15$)*	2**
7915	9	$B_9=3.52$	2**
7916	6	$B_6=2.12$	2**
7917	2	$B_2=1.48$	2**
8044	12	$B_{12}=5.3$	2**
8045	16	$B_{16}=-1.61$ ($Z=-.4025$)*	1*

* Since the sample size is large we used the fact that $B_n/\sqrt{n}=Z$ is asymptotically normal.
1* stands for not significant against H_1.
2** stands for not significant against H_2.

Table 4.

Plane	Sample size	Observed value of statistic A_n	Conclusions ($\alpha=.05$)
7908	23	-582.884 ($Z=-1.27$)*	2**
7909	29	-238.26 ($Z=-.54$)*	2**
7912	30	-394.55 ($Z=1.21$)*	2**
7913	27	396.3 ($Z=.993$)**	1*
7914	24	-8.97 ($Z=-.03$)*	2**
7917	2	-61.5	2**

* Since the asymptotic distribution of A_n depends on λ, we have used n/Y_n instead of λ.

Example 2. Consider the data given in Susarla and Van Ryzin [27]. The data represents the survival times of 81 participants from a melanoma study conducted by the central Oncology Group. For our study we have dropped censored observations, that is, we have 46 observa-

Table 5.

Statistics	Observed value	Conclusions ($\alpha=.05$)
A_{46}	984.4 ($Z=2.1$)	reject H_0 against the alternative HNBUE
B_{46}	-17.486 ($Z=-2.52$)	reject H_0 against the alternative HNBUE
$\frac{A_2\sqrt{7560}}{n^7}$	1.72	reject H_0 against the alternative IFR
$\frac{B\sqrt{210}}{n^5}$	1.93	reject H_0 against the alternative IFRA

tions. We want to test if the survival times follow exponential distribution or if they follow HNBUE. The computations associated with B_n and A_n are given in Table 5. Both A_n and B_n lead to rejection of H_0.

To check if H_0 will be rejected against more restricted alternatives like IFR or IFRA the statistics A_2 and B of Klefsjö [18] are also used in Table 5. Under H_0 Klefsjö showed that $A_2\sqrt{7560}/n^7$ and $B\sqrt{210}/n^5$ behave like standard normal distributions. A_2 and B also lead to rejection of H_0.

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