Ann. Inst. Statist. Math.
32 (1980), Part A, 111-120

AN ALTERNATIVE TO RATIO METHOD IN SAMPLE SURVEYS

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(Received Jan. 5, 1979; revised July 30, 1979)

Summary

This paper examines a simple transformation which enables the use of product method in place of ratio method. The convenience with the former, proposed by Murthy [3], is that expressions for bias and mean square error (mse) can be exactly evaluated. The optimum situation in the minimum mse sense and allowable departures from this optimum are indicated. The procedure requires a good guess of a certain parameter, which does not seem very restrictive for practice. Two methods for dealing with the bias of the estimator are mentioned. An extension to use multiauxiliary information is outlined.

1. Introduction

Consider a finite population with N units $U_1, \cdots, U_N$. The variate of interest y and the auxiliary variate x related to y assume real non-negative values (Y_i, X_i) on the unit U_i, $i=1, \cdots, N$. This non-negativity condition is met by almost all sample survey universes. $\hat{Y}$ and $\hat{X}$ are unbiased estimators of the parameters Y and X corresponding to the variates y and x respectively, based on any probability sampling design. Examples of such parameters are population totals and means. The value of X is assumed to be known. When the coefficient of correlation ρ between $\hat{Y}$ and $\hat{X}$ is positively high it is traditional to use the ratio method of estimation, where $\hat{Y}_r = \hat{Y}X/\hat{X}$ estimates Y. The bias and mse of $\hat{Y}_r$ are, upto second order moments

$$B(\hat{Y}_r) = Y(1-k)V_{02} , \tag{1.1}$$

$$M(\hat{Y}_r) = Y^2[V_{20} + (1-2k)V_{02}] , \tag{1.2}$$

where V_{ij} are the relative central moments defined by

Key words and phrases: Probability sampling design, ratio and product methods of estimation, bias, mean square error, interpenetrating subsamples, simple random sampling.
AMS subject classification: Primary 62 D05.

(1.3) $$V_{ij}=\mathrm{E}\,(\hat{Y}-Y)^i(\hat{X}-X)^j/Y^iX^j\,,$$

(1.4) $$k=V_{11}/V_{02}=\rho\sqrt{V_{20}/V_{02}}\,.$$

The closeness of the expressions (1.1), (1.2) respectively to the actual bias and mse of $\hat{Y}_r$ depends much on the composition of the population, the sampling design and the sample size. Hence these expressions must be taken with reservation (Murthy [4], p. 365).

2. Product method as alternative to ratio method

In contrast to the above, *exact* expressions for the bias and mse can be obtained if the product method of estimation is used. Motivated by this, consider simple transformations that render the situation suitable for a product method instead of the ratio method. For instance let $\hat{Z}=L-\hat{X}$, where L is a scalar to be chosen. Clearly $\hat{Z}$ is unbiased for $Z=L-X$ and $\mathrm{cor}\,(\hat{Y},\hat{Z})=-\rho$. Now consider the following estimator of Y:

(2.1) $$\hat{Y}_p=\hat{Y}\hat{Z}/Z\,.$$

Writing $\hat{Y}=Y(1+e_1)$, $\hat{Z}=Z(1+e_2)$ where $\mathrm{E}\,(e_1)=\mathrm{E}\,(e_2)=0$, we get the bias and mse of $\hat{Y}_p$, with $\theta=X/(L-X)$, as follows:

(2.2) $$B(\hat{Y}_p)=-\theta YV_{11}\,,$$

(2.3) $$M(\hat{Y}_p)=Y^2[V_{20}+\theta(\theta-2k)V_{02}+2\theta(\theta V_{12}-V_{21})+\theta^2V_{22}]\,.$$

The variance estimators for products of estimators have been considered by Goodman [1].

$M(\hat{Y}_p)$ is minimized when $\theta_{\mathrm{opt}}=(kV_{02}+V_{21})/(V_{02}+2V_{12}+V_{22})$ and the corresponding L is

(2.4) $$\begin{aligned}L_{\mathrm{opt}}&=(1+\theta_{\mathrm{opt}})X/\theta_{\mathrm{opt}}\\&=(1+1/k)X+(2V_{12}+V_{22}-V_{21}/k)X/(kV_{02}+V_{21})\,.\end{aligned}$$

Let $\hat{Y}_p^*$ denote $\hat{Y}_p$ for optimum L. The bias and mse of $\hat{Y}_p^*$ are given respectively by (2.2) and (2.3) with θ replaced by θ_{opt}. It is ideal to know L_{opt} so that $\hat{Y}_p^*$ can be computed. But in most surveys this is not the case, since $Y=\mathrm{E}\,(\hat{Y})$ is itself unknown, let alone the V_{ij}. At best an approximation to L_{opt} can be obtained.

For simple random or varying probability sampling with replacement or any scheme involving independent subsamples, the bias and mse of $\hat{Y}_p$ have the form

(2.5) $$B(\hat{Y}_p) = -\theta Y V'_{11}/n = -\theta Y \rho\sqrt{V'_{02}V'_{20}}/n ,$$

(2.6) $$M(\hat{Y}_p) = Y^2[\{V'_{20} + \theta(\theta - 2k)V'_{02}\}/n + 2\theta(\theta V'_{12} - V'_{21})/n^2 + \theta^2\{V'_{22} + (n-1)(V'_{20}V'_{02} + 2V'^2_{11})\}/n^3]$$

where V'_{ij} stands for V_{ij} in (1.3) for a sample of one unit or for one subsample and n is the sample size or the number of subsamples as the case may be. Expression (2.5) indicates that for large n the relative bias $B(\hat{Y}_p)/Y$ is likely to be negligible since $|B(\hat{Y}_p)|/Y \leqq \theta\sqrt{V'_{02}V'_{20}}/n$. Also by replacing V_{ij} in terms of V'_{ij} in (2.4) it is seen that L_{opt} and $(1+1/k)X$ have closely comparable magnitudes, the remaining term being usually negligible. The value of X is known, but the exact value of k is rarely available. However in repeated surveys or studies based on multiphase sampling, where information regarding the same variates is collected on several occasions, it is possible to guess accurately the values of certain parameters. This problem has been studied among others by Murthy ([4], pp. 96–99) and Reddy [8]. Hence we assume that k can be guessed. In turn a good approximation L_0 for L_{opt} can be obtained. We examine below to what extent L_0 may deviate from L_{opt} and yet give an estimator better than $\hat{Y}_r$ or $\hat{Y}$.

3. Allowable departures from optimum

To get tangible ideas, the V_{ij} with $i+j>2$ are ignored in the various expressions. Thus $\theta_{\text{opt}} = k$ and

(3.1) $$M(\hat{Y}_p^*) = (1-\rho^2)Y^2V_{20} = (1-\rho^2)\,V(\hat{Y}) ,$$

(3.2) $$M(\hat{Y}_p) = Y^2[V_{20} + \theta_0(\theta_0 - 2k)V_{02}] = M(\hat{Y}_p^*)[1 + \varepsilon^2\rho^2/(1-\rho^2)] ,$$

if $\theta = \theta_0 = k(1+\varepsilon)$ when $L = L_0$. It is seen that $M(\hat{Y}_p^*)$ is the same as the variance of the difference estimator $\hat{Y} - h(\hat{X} - X)$ in the ideal case, namely when h is the coefficient of regression of $\hat{Y}$ on $\hat{X}$. $M(\hat{Y}_p^*)$ is also the same as the large sample approximation to the mse of the regression estimator. From (3.2) it follows that the proportional increase in mse of $\hat{Y}_p$ over that of $\hat{Y}_p^*$ is less than α if

(3.3) $$|\varepsilon| < \sqrt{(1-\rho^2)\alpha/\rho^2} .$$

Thus to ensure only a small relative increase in mse, $|\varepsilon|$ must be close to 0 if ρ is high but can depart considerably from 0 if ρ is just moderate. Also from (1.2), (3.2), we get $M(\hat{Y}_r) - M(\hat{Y}_p) = Y^2[(k-1)^2 - (\theta_0 - k)^2]V_{02} > 0$ when

(3.4) $$\theta_0 \text{ lies between } (2k-1) \text{ and } 1 .$$

Similarly a necessary and sufficient condition for $M(\hat{Y}_p)<V(\hat{Y})$ is

$$0<\theta_0<2k\ . \tag{3.5}$$

To investigate where (3.4), (3.5) are satisfied simultaneously we distinguish between the cases $0\leqq k\leqq 1$ and $k>1$.

Case (i) $0\leqq k\leqq 1$

Here choose $L_0>2X$ so that θ_0 is in (0, 1). If k happens to be in (0, 0.5), condition (3.4) is automatically met since $2k-1<0$, but (3.5) needs

$$L_0>(1+1/2k)X\ . \tag{3.6}$$

On the other hand if k is in (0.5, 1), then (3.5) is always met since $2k>1$ but (3.4) requires

$$L_0<[1+1/(2k-1)]X\ . \tag{3.7}$$

Thus any $L_0>2X$ satisfying (3.6) or (3.7) as the case may be, will make $\hat{Y}_p$ an improved estimator.

Case (ii). $k>1$

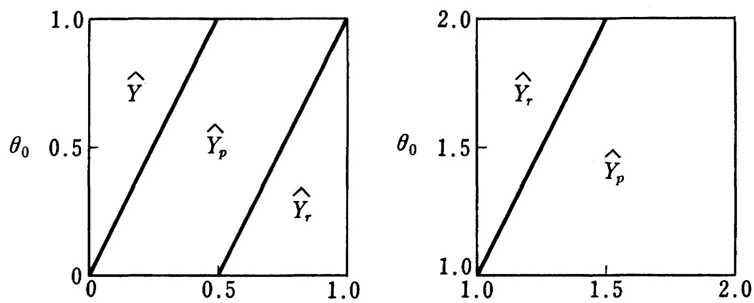
Here choose $L_0<2X$. In addition we need only that $L_0>[1+1/(2k$

Table 3.1 Optimum L and lower and upper bounds on the choice of L for typical values of k

k	Lower bound on L	Optimum L	Upper bound on L
(1)	(2)	(3)	(4)
0.1	$6.00X$	$11.00X$	∞
0.2	$3.50X$	$6.00X$	∞
0.3	$2.67X$	$4.33X$	∞
0.4	$2.25X$	$3.50X$	∞
0.5	$2.00X$	$3.00X$	∞
0.6	$2.00X$	$2.67X$	$6.00X$
0.7	$2.00X$	$2.43X$	$3.50X$
0.8	$2.00X$	$2.25X$	$2.66X$
0.9	$2.00X$	$2.11X$	$2.25X$
1.0	$2.00X$	$2.00X$	$2.00X$
1.1	$1.83X$	$1.91X$	$2.00X$
1.3	$1.63X$	$1.77X$	$2.00X$
1.5	$1.50X$	$1.67X$	$2.00X$
2.0	$1.33X$	$1.50X$	$2.00X$
2.5	$1.25X$	$1.40X$	$2.00X$
3.0	$1.20X$	$1.33X$	$2.00X$

$-1)]X$ for $\hat{Y}_p$ to be more precise than $\hat{Y}_r$ or $\hat{Y}$. To get a clearer idea some typical situations are presented in Table 3.1.

Interestingly the choice $L_0=2.25X$ covers a fairly wide range for k from 0.4 to 0.9, being actually optimum for $k=0.8$. Similarly $L_0=3.5X$ suits the range from 0.2 to 0.7 for k with optimum at $k=0.4$. In fact the choice of L_0 is very flexible when k is moderate, say in (0, 0.7). This flexibility disappears in the neighborhood of $k=1$. However a value like $L_0=1.9X$ is virtually safe for all $k>1$. Better selections can be made when k is known more precisely. Fig. 3.1 shows regions where $\hat{Y}_p$ is to be preferred.



Case(i) $0 \leqq k \leqq 1$ Case(ii) $k>1$

Fig. 3.1

4. Case of negative correlation

When $\rho<0$, take $\hat{Z}=L+\hat{X}$ so that $\text{cor}(\hat{Y}, \hat{Z})=\text{cor}(\hat{Y}, \hat{X})$. Here it is appropriate to compare $\hat{Y}_p$ with the traditional product estimator $\hat{Y}\hat{X}/X$. An approximation to L_{opt} is $-(1+1/k)X$. The restrictions on the choice L_0 for L_{opt} can be investigated. It turns out that $L_0=0.25X$ covers the range -0.9 to -0.4 for k, being the best at $k=-0.8$, while $L_0=1.5X$ is suitable for k in $(-0.7, -0.2)$ being actually optimum at $k=-0.4$. And a choice like $L_0=-0.10X$ is practically safe for all $k<-1$. A better selection can be made if k is more precisely known.

Table 4.1 Rules of thumb for choosing L

$k>0$		$k<0$	
Case	L	Case	L
(1)	(2)	(3)	(4)
$0<k\leqq 1/2$	$3.50X$	$-1/2\leqq k<0$	$1.50X$
$1/2<k<1$	$2.25X$	$-1<k<-1/2$	$0.25X$
$k>1$	$1.90X$	$k<-1$	$-0.10X$

The rules of thumb for choosing L when a firm guess of the value of k cannot be made, but only an interval containing k can be specified, are given in Table 4.1. These rules are not applicable when k is in the neighborhood of either 0 or ± 1. In fact if k is (i) close to 0 simple estimator $\hat{Y}$ is to be used, (ii) close to 1 either $\hat{Y}_r$ or $\hat{Y}_p$ with $L=2X$ may be used, and (iii) close to -1, $\hat{Y}_p$ with $L=0$ (which is the same as the usual product estimator) may be used.

5. Unbiased estimators

From (2.2) it is clear that $\hat{Y}_p$ is unbiased for Y when $\hat{Y}$ and $\hat{X}$ are uncorrelated. However this situation is not to be preferred since there will be an unacceptable increase in the variance as compared to $V(\hat{Y})$. The usable methods of dealing with the bias are outlined below. Method (i) is suited for any design using replicated samples. Method (ii) is appropriate for simple random sampling without replacement. Here the technique developed by Quenouille [6] and applied by Shukla [9] is used for making the estimator unbiased.

Method (i). Suppose the sample is in the form of n independent interpenetrating subsamples. Let $\hat{Y}_i$, $\hat{X}_i$ be unbiased estimators of Y and X based on ith subsample and $\text{cor}(\hat{Y}_i, \hat{X}_i)>0$, $i=1,\cdots,n$. Consider the following two product estimators of Y, with $\hat{Z}_i=L-\hat{X}_i$ and $Z=L-X$:

$$\hat{Y}_1=\left(\sum_1^n \hat{Y}_i/n\right)\left(\sum_1^n \hat{Z}_i/n\right)\Big/Z\,, \tag{5.1}$$

$$\hat{Y}_2=\sum_1^n \hat{Y}_i\hat{Z}_i/nZ\,. \tag{5.2}$$

As in Murthy [3], it can be shown that $B(\hat{Y}_2)=nB(\hat{Y}_1)$ and hence that $\hat{Y}_3=(n\hat{Y}_1-\hat{Y}_2)/(n-1)$ is unbiased for Y. The conditions for $\hat{Y}_3$ to be more precise than $\hat{Y}_1$ are similar to those given in Murthy and Nanjamma [2].

Method (ii). In the case of simple random sampling of $n=2m$ units without replacement, the sample may be split at random into two subsamples of m units each. Let Y, X be the population totals and $\hat{Y}_i$, $\hat{X}_i$, $i=1, 2$, be simple expansion estimators of Y and X based on the subsamples and $\hat{Y}$, $\hat{X}$ those based on the entire sample. Take $\hat{Z}_i=L-\hat{X}_i$, $i=1,2$; $\hat{Z}=L-\hat{X}$, $Z=L-X$. Here ρ is the same as $\text{cor}(y, x)$ and k reduces to $\rho C_y/C_x$ where C_y, C_x are the population cv's of y and x. Thus guessing k for choosing L is fairly easy especially when C_y,

C_x are of comparable magnitudes. A scatter diagram for at least a part of current data may help in this regard. Consider the product estimators

$$\hat{Y}_{pi}=\hat{Y}_i\hat{Z}_i/Z\ , \quad i=1, 2\ ; \qquad \hat{Y}_p=\hat{Y}\hat{Z}/Z$$

based on the two subsamples and the entire sample respectively. Then as in Shukla [9], it can be shown that an unbiased estimator of Y is

$$\hat{Y}_4=(2N-n)\hat{Y}_p/N-(N-n)(\hat{Y}_{p1}+\hat{Y}_{p2})/2N\ . \tag{5.3}$$

An expression for variance of $\hat{Y}_4$ is

$$\mathrm{V}(\hat{Y}_4)=Y^2[V_{20}+\theta(\theta-2k)V_{02}+(1-2w^2)(2\theta^2V_{12}-2\theta V_{21}+\theta^2V_{22})] \tag{5.4}$$

where $\theta=X/(L-X)$ as earlier and $w=-(N-n)/2N$. In this expression for variance the terms arising from subsampling have been retained only upto second order moments. This is expected to be satisfactory since the subsampling fraction is as large as 1/2. An estimate of $\mathrm{V}(\hat{Y}_4)$ is obtained by replacing Y by $\hat{Y}$ and V_{ij} by $v_{ij}=(1/n-1/N)\sum_{s=1}^{n}(Y_s-\bar{y})^i(X_s-\bar{x})^j/(n-1)\bar{y}^i\bar{x}^j$ in (5.4), where $\bar{y}$, $\bar{x}$ denote the sample means and $\sum_{s=1}^{n}$ denotes summation over the units in the sample.

6. Use of multiauxiliary information

Frequently information on several x-variates may be used; for instance utilizing census data to adjust current estimates. In this context an extension of the method of Sections 2 and 4 is as follows: Let $\hat{Y}$, $\hat{X}_t$ be unbiased estimators of the parameters Y, X_t, based on any sampling design; $t=1,\cdots,q+s$. All values are real, nonnegative and X_t are known. Also let $\mathrm{cor}(\hat{Y},\hat{X}_t)>0$ for $t=1,\cdots,q$ and $\mathrm{cor}(\hat{Y},\hat{X}_t)<0$ for $t=q+1,\cdots,q+s$. Then $\hat{Z}_t=L_t+\delta_t\hat{X}_t$ is unbiased for $Z_t=L_t+\delta_tX_t$ for each t. Take $\delta_t=-1$ for $t=1,\cdots,q$ and $\delta_t=1$ for $t=q+1,\cdots,q+s$. An estimator of Y is

$$\hat{Y}_p=\hat{Y}\sum_{1}^{q+s}W_t\hat{Z}_t/Z_t \tag{6.1}$$

where $W=(W_1,\cdots,W_{q+s})$, $\sum_{1}^{q+s}W_t=1$, is a vector of weights. Then

$$B(\hat{Y}_p)=Y\left(\sum_{q+1}^{q+s}W_t\theta_tV_{11}^{(t)}-\sum_{1}^{q}W_t\theta_tV_{11}^{(t)}\right) \tag{6.2}$$

and the mse, upto second order moments, is

$$M(\hat{Y}_p)=Y^2 \sum_{t,u=1}^{q+s} W_t W_u d_{tu}=Y^2\cdot WDW' \tag{6.3}$$

where the elements of the matrix D are given by

$$d_{tu}=V_{20}+\delta_t\theta_t V_{11}^{(t)}+\delta_u\theta_u V_{11}^{(u)}+\delta_t\delta_u\theta_t\theta_u V_{11}^{(tu)}$$

with $\theta_t=X_t/(L_t+\delta_t X_t)$, $t, u=1,\cdots,q+s$. Here $V_{ij}^{(t)}$ stands for V_{ij} given in (1.3) with $(\hat{X}-X)$ replaced by $\hat{X}_t-X_t$, X by X_t and $V_{11}^{(tu)}=\mathrm{E}(\hat{X}_t-X_t)(\hat{X}_u-X_u)/X_tX_u$. As shown in Rao and Mudholkar [7] the matrix D is positive definite if the $(q+s+1)\times(q+s+1)$ matrix of the cv's of $\hat{Y}$ and $\hat{Z}_t$ is positive definite.

Theoretically the L_t can be determined to minimize $M(\hat{Y}_p)$. But a practicable alternative is to choose L_t such that d_{tt} is controlled. Thus, as a rule of thumb, L_t may be $3.5X_t$ if $k_t=V_{11}^{(t)}/V_{02}^{(t)}$ is positive but moderate, while L_t may be $1.5X_t$ when k_t is negative but moderate. Other choices can be made as discussed in Sections 3 and 4. In any case a scatter diagram of y against each x-variate values in the sample may be helpful.

Next, applying the generalized Cauchy inequality (see Olkin [5]), the W_t optimum in the sense of minimizing $M(\hat{Y}_p)$ for given D are provided by $W_{\text{opt}}=eD^{-1}/eD^{-1}e'$ where $e=(1,\cdots,1)$. Substituting W_{opt} in (6.3), $M_{\min}(\hat{Y}_p)=Y^2/eD^{-1}e'$. However, in surveys W_{opt} can rarely be computed and used since the matrix D is usually unknown. Theoretically W_t will all be equal $(=1/q+s)$ if and only if the column sums of D are equal. A hypothetical example of this occurs when the population cv's of the $\hat{Z}_t$ are all equal, $\hat{Y}$ is equally correlated with all $\hat{Z}_t$ and all pairs of two different estimators $\hat{Z}_t$ have the same correlation. Usually the W_t are selected from experience and theoretical considerations. In small scale surveys of specialized scope it may be feasible to estimate W_{opt} from the sample itself.

7. Empirical performance of $\hat{Y}_p$

For purposes of illustration, simple random sampling without replacement is assumed throughout this section and Y, X denote the population totals. The following four populations are considered.

Table 7.1 Values of y and x for population 1

y	1	2	4	5	6	8	9
x	6	5	7	2	4	10	8

Population 3. Data on number of workers (x_1), fixed capital (x_2) and output (y) for 80 factories in a certain region ([4], p. 228).

Table 7.2 Values of y, x_1 and x_2 for population 2

y	12	22	38	15	18	31	15	20	10	25	11	17	12	22	14	26	8	16	13	19
x_1	14	25	37	18	20	30	15	21	12	28	14	19	12	23	16	28	9	15	15	20
x_2	30	25	9	30	28	12	30	24	36	28	30	30	31	25	31	25	35	25	30	28

Population 4. Data on cultivated area (y) and area under wheat (x_1, x_2) during two different years for 34 villages in a certain region ([4], p. 399, Table 10.6).

For populations 2, 3 and 4, three subcases are studied: using x_1 alone, x_2 alone, and x_1, x_2 as auxiliary variates. These are respectively denoted by 2a, 2b, 2c etc. In the case of population 1, all possible samples of $n=3$ units were listed and the biases and mse's were computed from first principles to avoid any approximations, while in 2a the exact expressions for bias and mse were used. In the remaining cases computations were made only upto second order moments. The rules of thumb in Table 4.1 were applied for choosing L for $\hat{Y}_p$, while L was taken to be $(1+1/k)X$ or $-(1+1/k)X$ as the case may be, for $\hat{Y}_p^*$. When information on x_1 and x_2 was utilized, $\hat{Y}_p$ was compared with the generalized multivariate estimator discussed in [7], with weights $W_1=W_2=1/2$. The results are summarized in Table 7.3. For simple random sampling without replacement the relative bias of the suggested estimator $\hat{Y}_p$ reduces to $B(\hat{Y}_p)/Y=-((N-n)/Nn)\theta S_{11}/\bar{Y}\bar{X}$, where S_{11} is the population covariance between y and x, and $\bar{Y}$, $\bar{X}$ are population means. Hence

Table 7.3 Efficiency and relative bias of different estimators

Population	k	Estimator			
		$\hat{Y}$	Ratio (product) estimator	$\hat{Y}_p$	$\hat{Y}_p^*$
(1)	(2)	(3)	(4)	(5)	(6)
1	0.57	100	94 (10)	115 (9)	121 (6)
2 a	1.11	100	2433 (1)	4253 (16)	4253 (16)
2 b	−0.50	100	100 (3)	109 (1)	109 (1)
2 c	1.11, −0.50	100	218 (2)	315 (1)	798 (1)
3 a	0.35	100	32 (58)	729 (13)	837 (11)
3 b	0.44	100	65 (33)	1088 (10)	1197 (11)
3 c	0.35, 0.44	100	45 (46)	246 (11)	1052 (11)
4 a	0.75	100	376 (13)	565 (31)	577 (29)
4 b	0.71	100	318 (16)	514 (32)	549 (29)
4 c	0.75, 0.71	100	365 (15)	570 (32)	590 (29)

$$[Nn/(N-n)]\cdot|B(\hat{Y}_p)|/Y=\theta S_{11}/\bar{Y}\bar{X}\ ,$$

where the right side expression is seen to be independent of the sample size n. To have this practical convenience, values of $100[Nn/(N-n)]\cdot|\text{Bias}|/Y$ are reported in Table 7.3 within parentheses.

The value of k ranges from -0.50 to 1.11 in Table 7.3. Here substantial gain in efficiency is seen when $\hat{Y}_p$ is used instead of the traditional estimators, in most of the cases. Also $\hat{Y}_p$ compares quite well with the ideal case of $\hat{Y}_p^*$. Thus the illustrations indicate that (i) the use of $\hat{Y}_p$ is desirable in practice, and (ii) the rules of thumb for choosing L work well. If an unbiased estimator is preferred then the techniques outlined in Section 5 may be employed.

Acknowledgement

The authors acknowledge the valuable comments by the referee.

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