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NONEXISTENCE OF ESTIMATES WHICH MINIMIZE $x'V^{-1}x$ IN AN EXPONENTIAL TYPE OF STATIONARY TIME SERIES

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In estimating the structural parameters $\boldsymbol{\theta}=(\theta_1,\cdots,\theta_n)'$ of a spectral density $f(\lambda)=(\sigma^2/2\pi)g(\lambda|\boldsymbol{\theta})$ of a stationary time series one often (e.g. [2], [3]) uses the estimates which minimize $\boldsymbol{x}'V^{-1}(\boldsymbol{\theta})\boldsymbol{x}$ where $\boldsymbol{x}=(x_1,\cdots,x_n)'$ is a sample vector and $\sigma^2V(\boldsymbol{\theta})$ is the variance matrix. The purpose of this paper is to show that there are, unless $\boldsymbol{x}=\boldsymbol{0}$, no such estimates in the exponential model of spectrum

$$f(\lambda)=\frac{\sigma^2}{2\pi}\exp\left(\sum_{s=1}^{p}\theta_s\cos s\lambda\right)$$

which was proposed for fitting a stationary time series by Bloomfield [1]. This fact directly follows from the following theorem.

THEOREM. *In the exponential type, for any fixed sample* $\boldsymbol{x}=(x_1,\cdots,x_n)'$ *there is a sequence* $\boldsymbol{\theta}_j$ *such that* $\lim_{j\to\infty}\boldsymbol{x}'V^{-1}(\boldsymbol{\theta}_j)\boldsymbol{x}=0$.

PROOF. Let's consider the case that $\theta_1>0$ and $\theta_j=0$ for $j\geqq 2$. Then, the minimum eigenvalue of $V(\boldsymbol{\theta})$ is

$$\min_{\boldsymbol{\xi}'\boldsymbol{\xi}=1}\boldsymbol{\xi}'V(\boldsymbol{\theta})\boldsymbol{\xi}=\frac{1}{2\pi}\min_{\boldsymbol{\xi}'\boldsymbol{\xi}=1}\int_{-\pi}^{\pi}\left|\sum_{t=1}^{n}\xi_t e^{it\lambda}\right|^2\exp(\theta_1\cos\lambda)d\lambda$$
$$\geqq\frac{1}{2\pi}\exp\left(\frac{1}{2}\theta_1\right)\min_{\boldsymbol{\xi}'\boldsymbol{\xi}=1}\int_{-\pi/3}^{\pi/3}\left|\sum_{t=1}^{n}\xi_t e^{it\lambda}\right|^2 d\lambda\,.$$

It is easily found that $\min_{\boldsymbol{\xi}'\boldsymbol{\xi}=1}\int_{-\pi/3}^{\pi/3}\left|\sum_{t=1}^{n}\xi_t e^{it\lambda}\right|^2 d\lambda$ is positive. Hence, $\lim_{\theta_1\to\infty}\min_{\boldsymbol{\xi}'\boldsymbol{\xi}=1}\boldsymbol{\xi}'V(\boldsymbol{\theta})\boldsymbol{\xi}=\infty$. This completes the theorem. Q.E.D.

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