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A RANDOM PACKING MODEL FOR ELECTIONS

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1. Introduction

The object of this paper is to introduce a 1-dimensional random closest packing model for elections to explain relative abundances of polls among candidates nominated by a political party such as the Liberal Democratic Party or the Socialist Party in Japan. 1-dimensional random packing is known as car-parking problem. Our conclusion is that the nomination of candidates by a certain political party seems to be often performed like car-parking on a street. The length of the street corresponds to the total polls obtained by the party. A car corresponds to a candidate. The distance from the top of a car to the top of the car just behind corresponds to the number of polls obtained by the candidates. For model fitting, we consider random packing on a circle to eliminate the influence of the ends.

Several authors made stochastic treatments on election. Kakeya [9] discussed a stochastic model on a final election vote. Aoyama [1] considered a random division model to explain the relative abundances of polls among candidates. (For related works see also the papers of mathematical ecology, Shinozaki and Urata [13] and MacArthur [10].) Hayashi and Takakura [7] discussed how to predict results of an election from surveys before the election. (See also Hayashi [6].)

Observations and data on elections in Japan are in Nisihira [11]. We can also find the data in the press, for example, Yomiuri, Asahi, and Mainichi, which have several millions of circulation. Our study is based on Nisihira [11] and after 1972 on Asahi. Our model explains many data which are not clearly explained by random division.

We consider elections for the House of Councillors at national constituency. The parliament of Japan consists of the House of Representatives and the House of Councillors. The members of the House of Councillors are devided into those elected at a national constituency and the members who are elected at local constituencies. The fixed number of the members of the House of Councillors is 252 persons, out of which 100 are elected by universal votes at the national constituency, that is to say, at the whole country as one unit. The election is carried out for each three years. The half of the 100 persons are elected at each election at national constituency. Hence the member can stay in the parliament for six years without passing through an election. At the national constituency, candidates who gain not less than the 50th largest polls are elected. There are several political parties in Japan, for example Liberal Democratic Party, Socialist Party and Communist Party. For these twenty years L.D.P. (Liberal Democratic Party) has the largest share in the House of Councillors and S.P. (Socialist Party) has the second largest. Each political party intends to get as possible as many candidates elected. So the most succesful case for a party is that all candidates of the party gain the minimum polls necessary to be elected. But this does not happen usually. The control by L.D.P. on organizations which support L.D.P. are restricted, since each organization supports persons who represent its profit and does not support the other persons. Under such restrictions, the random packing may be a good solution for L.D.P. to nominate candidates at national constituency.

The procedure of the nomination is supposed to be a random packing on a circle of unit length by arcs with length α. Let us write $N(\alpha)$ as a random variable, which represents the number of the arcs in random closest packing. Then the expectation of the packing density approaches to $\lambda \doteqdot 0.748$ as α approaches to zero (Rényi [12]). The corresponding central limit theorem is proved by Dvoretzky and Robbins [4]. The distribution of length of gaps generated by sticks packed randomly is discussed by Bankövi [2].

For two or more than two dimension mathematical treatment is very difficult and most of the previous works are based on experiments. One of interesting applications of two-dimensional random packing is given by Hasegawa and Tanemura [5]. They explained the shape of a territory of a species of birds "calidris melanotos" by random packing. In our previous paper [8], we discussed a random packing model for election on the data of 1977 and 1974 for L.D.P., in which $m \leqq N(\alpha)$ is assumed. Here we apply our random closest packing model, in which $N(\alpha)=m$ is assumed, not only to L.D.P. but also to S.P. For S.P., from 1956 till 1968, and for L.D.P., from 1965 till 1974, candidates seem to be nominated as the way in our model.

2. Random closest packing model

We assume all polls for L.D.P. of a district can be numbered from 1 to x. L.D.P. gives an order by priority to each person, who can organize d polls out of the x polls, as $c_1, c_2, c_3, \cdots$. Consider x points on a unit circle $(0, 2\pi]$, $\frac{1}{x}2\pi$, $\frac{2}{x}2\pi$, $\cdots$, $\frac{x}{x}2\pi$. The points on the arc $\left[\frac{X_i}{x}2\pi, \frac{X_i+d}{x}2\pi\right)$ are assumed to be the territory of c_i, where the arc $\left[\frac{X_i}{x}2\pi, \frac{X_i+d}{x}2\pi\right)$ represents the shorter arc of the circle with the ends $\frac{X_i}{x}2\pi$ and $\frac{X_i+d-1}{x}2\pi$ for $d/x<1/2$. X_i is placed at random by the uniform distribution on the circle and each of the X's is mutually independently placed for $i=1, 2, 3, \cdots$. L.D.P. nominates c_1 at first on the circle. For $k\geqq 2$, if the territory of c_k does not overlap a territory of a person nominated previously, L.D.P. nominates c_k, otherwise L.D.P. does not nominate him. L.D.P. continues to nominates candidates until there is no empty arc$\geqq\frac{d}{x}2\pi$. The number of arcs packed by this process is represented by a random variable $M(d)$. In this way candidates $c_{i_1}, c_{i_2}, \cdots, c_{i_{M(d)}}$ are nominated. We rearrange the c's as $c_{j_1}, c_{j_2}, \cdots, c_{j_{M(d)}}$ so as to be

$$0<X_{j_1}<X_{j_2}<\cdots<X_{j_{M(d)}}\leqq x\,.$$

We assume L.D.P. assigns the polls on the arc $\left[\frac{X_{j_k}+d}{x}2\pi, \frac{X_{j_{k+1}}}{x}2\pi\right)$ to c_{j_k} where $X_{j_{M(d)+1}}=X_{j_1}$. Hence the polls obtained by the candidate c_{j_k} are the points on the arc $\left[\frac{X_{j_k}}{x}2\pi, \frac{X_{j_{k+1}}}{x}2\pi\right)$.

3. Applications

We approximate the above discrete random packing by a continuous one on a circle of unit length by arcs of length α. Let the number of arcs with length α packed on the circle be $N(\alpha)$, we have $\lim_{\alpha\to 0} \mathrm{E}\,(\alpha N(\alpha))=\lambda\fallingdotseq 0.748$. (See Rényi [12].) Denoting the length of the i-th longest arc under the condition $N(\alpha)=m$ with $m\alpha=\lambda$, by a random variable Z_i, we define $P_i(\alpha, m)$ by

$$P_i(\alpha, m)=\mathrm{E}\,(Z_i|N(\alpha)=m)\,.$$

If the nomination of candidates is performed as the way in our model, $P_i(\alpha, m)$ will fit the data. We use the value $\frac{1}{100}\sum_{i=1}^{100} Z_{ij}$, obtained by computer experiments, for $P_i(\alpha, m)$ where Z_{ij} is the number of points on the i-th longest arc of the j-th experiment in which $N(\alpha)=m$ with $m\alpha=0.748$. An estimation of p. 219 in Dvoretzky and Robbins [4] justifies that the assumption $m\alpha=0.748$ is reasonable. Their estimation shows

$$|\mathrm{E}\,(\alpha N(\alpha))-\lambda|<\frac{2^n}{n!}$$

for the integer n which satisfies $\frac{1}{n+2}\leqq\alpha\leqq\frac{1}{n+1}$.

Before treating the data, we must mention that there is a group of candidates who have not organized polls, but are well-known by mass media. For example, talents of television and writers are included in it. Candidates of this group is called Talented Candidates. L.D.P. nominates a part of the group. But their way of getting polls is completely different from the way of candidates who are based on organized polls. Talented Candidates concern to floating polls not to organized polls. So we exclude the Talented Candidates for our model fitting. Consider the election in 1974. In fig. 1, the real line represents gains of 28 candidates of L.D.P. (7 of Talented Candidates are excluded), in order of descending magnitude, where total gains obtained by the 28 candidates are considered to be 1, and the dotted line represents $P_i(\alpha, m)$ with $m=28$ and $\alpha=0.748/m$. In fig. 2, the real line represents the gains obtained by each of 10 Talented Candidates and the dotted line represents

$$\frac{1}{m}\sum_{r=1}^{m-i+1}\frac{1}{m-r+1}, \qquad \text{with } m=10\,,$$

which is the expected length of ordered random intervals given in Barton and David [3]. The results of 1977, 1971, 1968 and 1965 are in fig. 3, fig. 4, fig. 5 and fig. 6 respectively. Our model does not seem to fit the data of 1968 and 1971. The reason may be that L.D.P. nominated candidates, who had no possibility to be elected, by some political strategy or consideration, such as to preserve a balance among factions or to strengthen supports for L.D.P. for next elections. If we exclude such candidates to consider an appropriate m, our model will fit the data.

Our model explains the data of Socialist Party in 1968, 1965, 1962 and 1956 as shown in fig. 7, fig. 8, fig. 9 and fig. 10.

We stress that the results are explained by only one parameter m, since α is determined by $m\alpha=0.748$.

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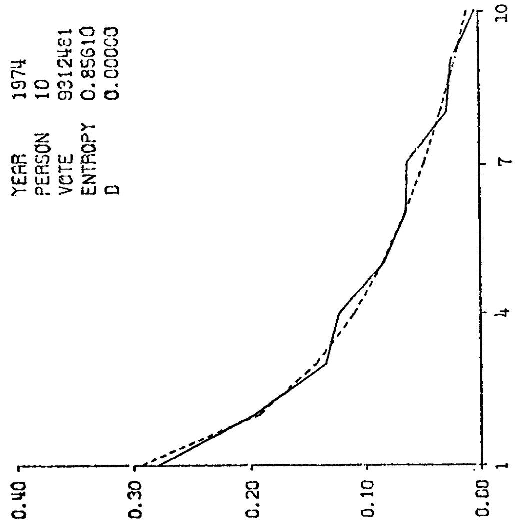


Fig. 2

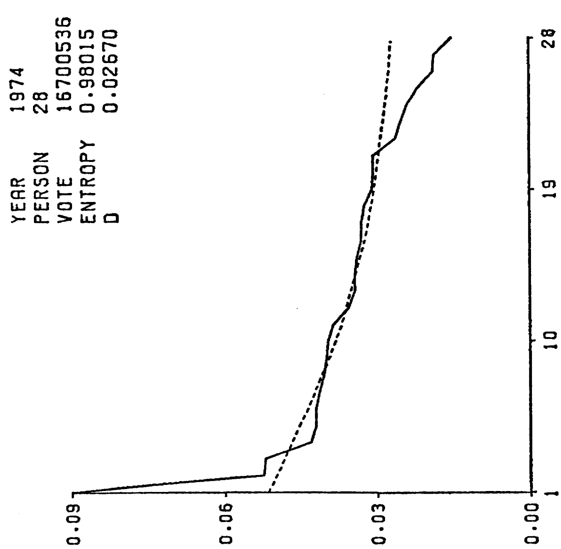


Fig. 1 7 of G.M.M. are excluded

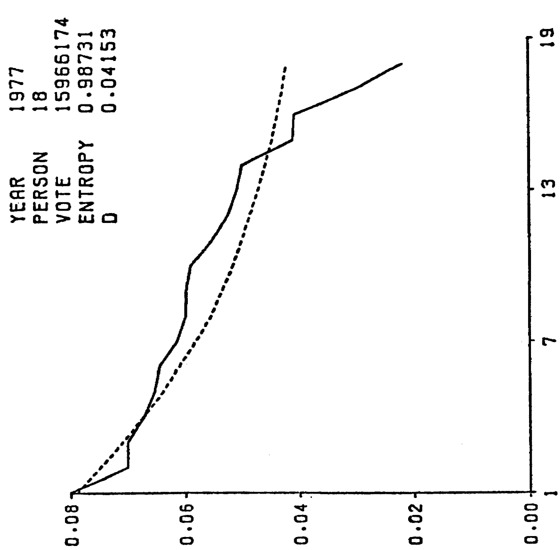


Fig. 3 4 of G.M.M. are excluded

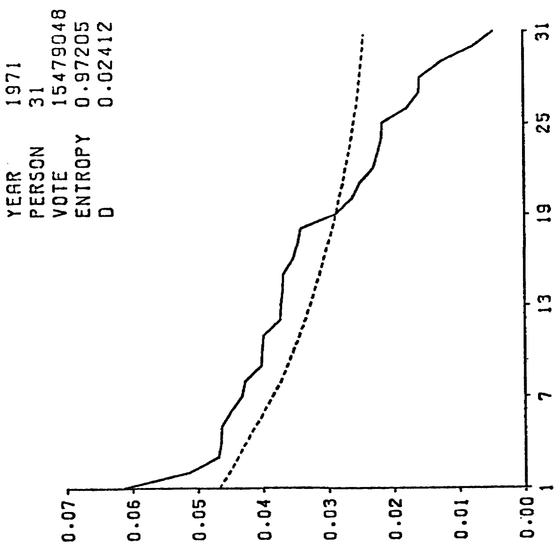
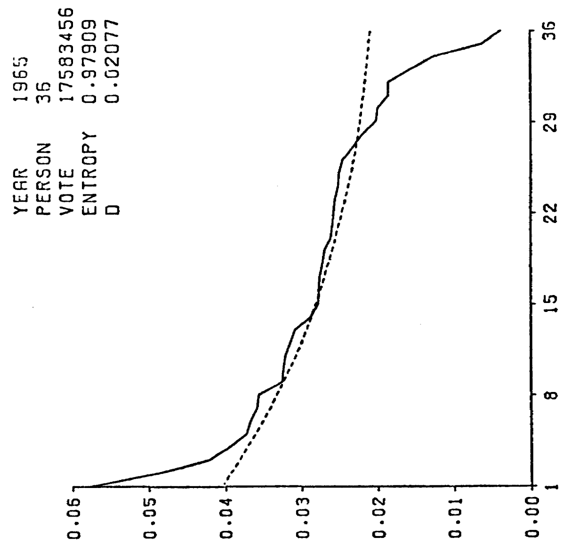


Fig. 4 3 of G.M.M. are excluded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PERSON 36
VOTE 17583456
ENTROPY 0.97909
D 0.02077
0.06
0.05
0.04
0.03
0.02
0.01
0.00
1
8
15
22
29
36

Fig. 6

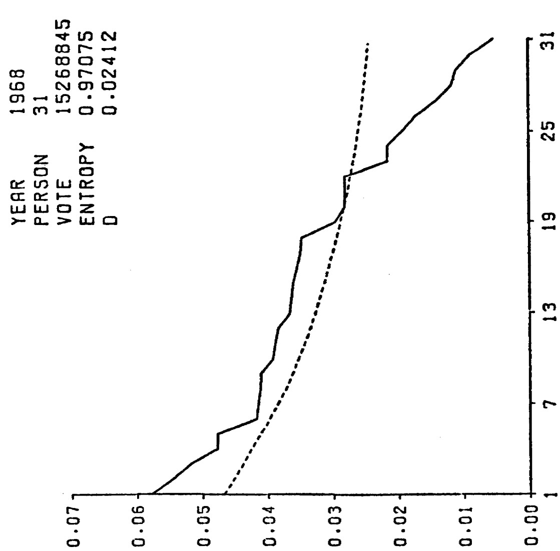
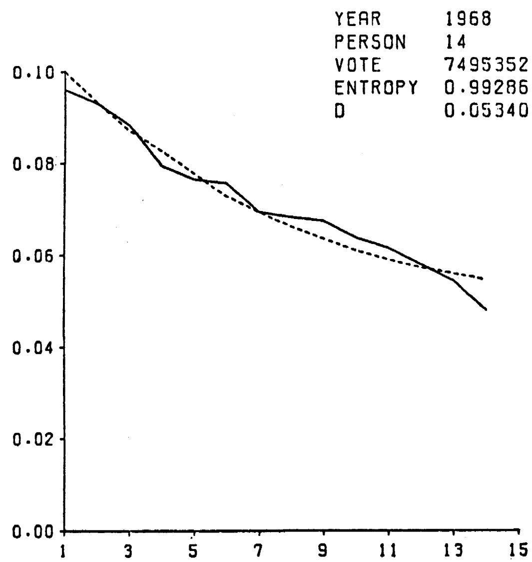

YEAR 1968
PERSON 31
VOTE 15268845
ENTROPY 0.97075
D 0.02412
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
1
7
13
19
25
31

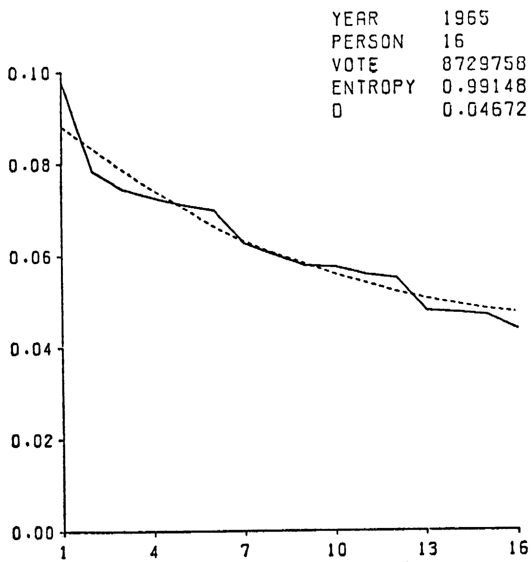
Fig. 5



PERSON= 14 D=0.05340

	VOTE	VOTE/SUM	RANDOM PACKING
1	720624	0.0961428	0.1000711
2	698065	0.0931330	0.0932144
3	662474	0.0883846	0.0870951
4	596371	0.0795654	0.0826448
5	574031	0.0765849	0.0777348
6	567015	0.0756488	0.0727332
7	520494	0.0694422	0.0693542
8	511587	0.0682539	0.0661604
9	505332	0.0674194	0.0634782
10	477498	0.0637059	0.0609377
11	461500	0.0615715	0.0589240
12	433856	0.0578833	0.0571844
13	407635	0.0543850	0.0558989
14	358870	0.0478790	0.0545629
	7495352	0.9999997	0.9999943

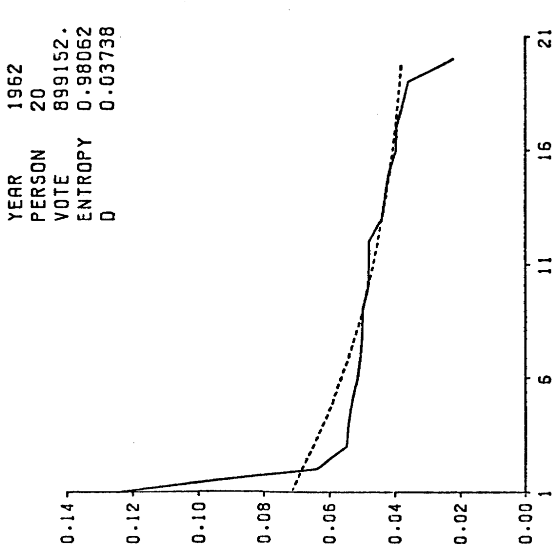
Fig. 7



PERSON= 16 D=0.04672

	VOTE	VOTE/SUM	RANDOM PACKING
1	854272	0.0978574	0.0881844
2	685280	0.0784993	0.0834669
3	650828	0.0745528	0.0786521
4	634408	0.0726718	0.0742034
5	620193	0.0710435	0.0704163
6	610493	0.0699324	0.0663450
7	548022	0.0627763	0.0630291
8	526247	0.0602820	0.0603334
9	505048	0.0578536	0.0582362
10	502200	0.0575274	0.0558196
11	487622	0.0558574	0.0539518
12	480667	0.0550607	0.0520139
13	418500	0.0479395	0.0504946
14	414701	0.0475043	0.0492777
15	409457	0.0469036	0.0481426
16	381820	0.0437378	0.0474266
	8729758	0.9999997	0.9999936

Fig. 8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PERSON 20
VOTE 899152.
ENTROPY 0.98062
D 0.03738
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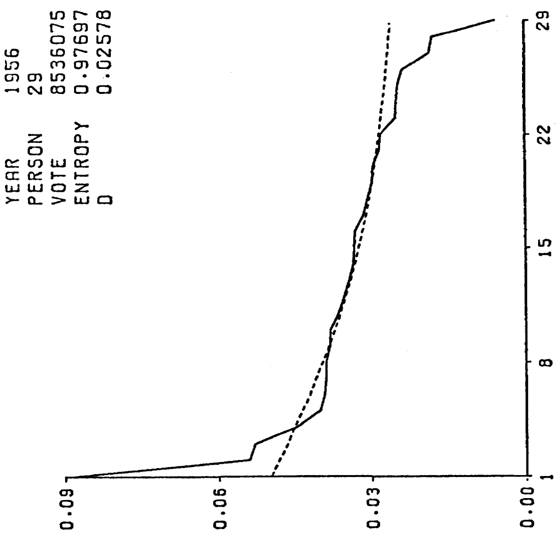
0.12
0.10
0.08
0.06
0.04
0.02
0.00
1
6
11
16
21

Fig. 9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PERSON 29
VOTE 8536075
ENTROPY 0.97697
D 0.02578
0.09
0.06
0.03
0.00
1
8
15
22
29

Fig. 10