ON AN ANALYSIS OF NATURAL DISASTER ON THE RAILWAY*

HIROJIRO AOYAMA

1. Introduction

Natural disaster on the railway such as landslip, falling stones, flood, snow and earthquake causes every year a great deal of losses in human life and railway equipments in Japan. In 1962 a large scale landslip occurred near Toyonaga in Dosan-line, Shikoku Island and the transportation between Keihanshin District and Shikoku Island was stopped about thirty days. Consequently we suffered from heavy economic losses. Therefore Japanese National Railways made a Committee of Countermeasures against Disaster and many technicians and scholars inside and outside the institution were invited to join this committee.

In this paper the author intends to describe briefly an analysis of landslip disaster. A landslip is often caused by rain whether in case of the large amount of continuous rain or heavy hourly rainfall, so that we must at first analyze the distribution of rainfall and then the effect of the other conditions in the site of disaster.

We considered for this purpose the other conditions as follows:

1. protection equipment or natural feature
2. gradient of protection equipment or natural slope
3. nature of soil
4. geological feature.

All trackways from Awa-Ikeda to Amatsubo in Dosan-line were investigated from this standpoint.

2. Analysis of rainfall

There were five observation spots of rainfall in this area and their data by self-registering rain gauges were analyzed. We defined the con-

---

* This paper is a part of statistical study which was treated by the Committee of Countermeasures against Disaster sponsored by Japanese National Railways. The study of the last section was partly carried out by the grant for scientific study in the Ministry of Education.

tinuous rainfall as the amount of a rainfall which is separated from others by the interruption longer than one hour. Assuming that the continuous rainfalls in five areas follow the Gamma distributions, we estimated the frequency of 0 mm rainfall, which means such a small amount of rainfalls as can not be registered in our apparatus, by the minimum Chi-square method.

Then the relation giving an estimate of daily rainfall from continuous rainfall, and that of hourly one from continuous one were obtained by usual regression equations. This latter relation can be used for the criterion to order patrol. Some figures in these relations are shown in Table 1.

Table 1. Relations between continuous rainfall and hourly rainfall in five areas.

Continuous rainfall (mm)	Hourly rainfall (mm)				
	Kawaguchi	Ōboke	Ōtaguchi	Ōsugi	Amatsubo
25	9.7	8.4	8.3	8.8	9.1
50	19.0	15.8	15.3	17.0	17.3
75	28.2	23.3	22.4	25.2	25.4
100	37.5	30.4	29.4	33.5	33.6
150	56.0	45.5	42.0	49.8	49.8
200	74.5	60.3	55.9	66.4	66.1

3. Analysis of disaster occurrence rate

The whole trackways from Awa-Ikeda to Amatsubo were divided into units of local areas which are classified by four factors—sort of protection equipments, its steepest gradient, nature of soil and geological feature in the case of unit with protection equipments. For units with any alteration of equipments we consider the original patterns as well as the present. Thus we obtained 169 different kinds of units with the equipments.

In a similar way we had 177 different kinds of units without equipments. The reason we treat separately these two cases is that the unit with equipments may originally have been more dangerous than the one without equipments.

As the distributions of rainfall in five observation spots are different from each other, weighting by the number of rainfalls to get a unified value in the whole area, we got the disaster occurrence rate (d.o.r.) in each unit, that is, the disaster occurrence frequency per year, per kilometer, and per number of rainfalls in each unit.

Let this d.o.r. be p_{ijkl}, where subscripts denote the ith equipment, the jth steepest gradient, the kth nature of soil and the lth geological feature. Putting

$$p_{ijkl}=x_i+y_j+z_k+u_l+w_{ij}+\varepsilon_{ijkl}\,,$$

where x_i, y_j, z_k, u_l, w_{ij} and ε_{ijkl} are the effect of the *i*th equipment, that of the *j*th steepest gradient, that of the *k*th nature of soil, that of the *l*th geological feature, the interaction effect of the (i, j)th between the equipment and steepest gradient and the effect of the error term, respectively, we obtained the solution by the least square method from the simultaneous equations:

$$f_{i...}x_i+\sum_j f_{ij..}y_j+\sum_k f_{i.k.}z_k+\sum_l f_{i..l}u_l+\sum_j f_{ij..}w_{ij}=\sum_{j,k,l} f_{ijkl}p_{ijkl} \text{ etc.}$$

$$f_{ij..}x_i+f_{ij..}y_j+\sum_k f_{ijk.}z_k+\sum_l f_{ij.l}u_l+f_{ij..}w_{ij}=\sum_{k,l} f_{ijkl}p_{ijkl} \text{ etc.}$$

where f_{ijkl} is 1 or 0 according as the corresponding unit exists or not,

Table 2.

Protection equipment	x
1. Revetment	27.30
2. Retaining wall	0
3. Piling stones	5.21
4. Covering with concrete	1.60
5. Covering with stones	2.90
6. Cover against falling stones	3.52
7. Revetment for falling stones	2.55
8. Stockade protecting falling stones	7.27
9. Others	1.49
Standard deviation of units in regard to x	5.38

Table 3.

Steepest gradient (cotangent)	y
0. 0 ~0.24	6.32
1. 0.25~0.49	5.94
2. 0.50~0.74	4.98
3. 0.75~0.99	0
4. 1.00~1.49	7.83
5. 1.50~1.99	5.71
9. 5.00~5.99	5.71
L. Level	3.69
S.D.	2.35

Table 4.

Nature of soil	z
1. Gravel	1.30
2. Sand	0
3. Hard rock	−0.59
4. Brittle rock	1.22
5. Brittle rock mixed with sand	−0.78
S.D.	0.46

Table 5.

Geological feature	*u*
1. Vein clayslate and graphite schist	1.67
2. Sand stone and sandy schist	2.00
3. Pebble schist	8.54
4. Lime stone and crystal lime	0
5. Chert and quartz-schist	−0.34
6. Schalstein and chlorite-schist	1.25
7. Diabase	−0.32
8. Pebble stone	−1.57
9. Strongly exfoliated schist (sandy schist)	0.41
10. Strongly exfoliated schist (graphite schist)	0.97
S.D.	1.30

Table 6.

Interaction (equipment × steepest gradient)	*w*
1×0*	−29.54
1×1	−35.27
1×4	−37.32
2×0	− 3.84
2×1	− 7.16
2×2	0.17
2×3	− 1.90
2×L	− 6.79
3×0	−13.72
3×1	−14.20
3×2	−13.13
3×4	−13.96
3×5	−13.82
4×0	− 6.11
4×1	− 8.29
4×2	− 7.79
4×3	− 2.80
4×4	−11.15
4×5	−10.20
4×L	− 7.38
5×1	−11.90
5×2	−10.24
5×3	− 4.02
5×4	−10.50
5×L	− 7.57
6×0	−12.28
6×1	−12.16
6×2	−11.73
6×3	− 6.74
6×5	−18.95
6×L	− 8.09
7×0	− 9.39
7×1	0.94
7×4	−11.26
8×0	−15.92
8×1	−15.54
8×2	−14.77
8×3	− 9.31
8×4	−13.20
8×5	−15.24
8×L	−11.84
9×1	−10.20
9×2	− 7.88
9×3	− 4.38
9×4	−11.69
9×L	− 4.70
S.D.	6.43

* 1×0 means the interaction effect between equipment 1. (revetment) and steepest gradient 0. (0∼0.24).

and $f_{ij\cdot\cdot}$ is the frequency of the (i, j) units. In these simultaneous equations some of variables are not clearly determined because of relations among them, and $x_i+y_j+w_{ij}$ may only be determined as a whole. However, changing the variable which can be taken as arbitrary, we can estimate each effect. The result is shown in Tables 2 to 6.

Here we can interpret the effect of the interaction term as the designed strength.

Whether we should take the interaction term into account depends upon the similarity between the estimated d.o.r. and real ones. Table 7 shows this relation in case the interaction is taken into account.

Table 7.

Actual d.o.r. / Estimated d.o.r.*	0	$\bar{0}$~	2~	4~	6~	8~	10~	12~	14~	20~	Total
10~	42										42
12~	82	5	3		1						91
14~	8	3	5	1	2	1		1		1	22
16~	3	1			1		1		1		7
18~		1			1		1			1	4
20~	2							1			3
Total	137	10	8	1	5	1	2	2	1	2	169

* Estimated values added by 12.11.

$\bar{0}$ indicates not including 0.

In this case we calculated the standard deviation (2.16) of ε_{ijkl} and compared it with those of x_i, y_j, w_{ij}, z_k and u_l. This result indicates that the effects of protection equipments and its interaction with the steepest gradient are larger than the error term's S.D., the effect of the steepest gradient is comparable with the error term's S.D., and those of geological feature and of nature of soil are not significant.

Similar analysis of the units without equipments was carried out, and the results are shown in Tables 8 to 13.

Table 8.

Natural feature	x
1. Protection forest	0
2. Natural slope (soil)	3.60
3. Natural slope (mountain mass)	9.89
4. Others	2.61
S.D.	3.55

Table 9.

Steepest gradient	y
0. 0 ~0.24	4.31
1. 0.25~0.49	4.31
2. 0.50~0.74	0.61
3. 0.75~0.99	0
4. 1.00~1.49	1.94
5. 1.50~1.99	1.23
6. 2.00~2.99	1.63
7. 3.00~3.99	2.05
8. 4.00~4.99	0.50
9. 5.00~5.99	2.38
10. 6.00~	2.13
L. Level	1.88
S.D.	1.29

Table 10.

Nature of soil	z
1. Gravel	0.45
2. Sand	0
3. Hard rock	1.34
4. Brittle rock	1.10
5. Brittle rock mixed with sand	9.99
S.D.	1.04

Table 11.

Geological feature	u
1. Vein clayslate and graphite schist	−0.40
2. Sand stone and sandy schist	2.28
3. Pebble schist	0.09
4. Lime stone and crystal lime	0
5. Chert and quartz-schist	−0.45
6. Schalstein and chlorite-schist	−1.64
7. Diabase	0.05
8. Pebble stone	−0.63
9. Strongly exfoliated schist (sandy schist)	−1.10
10. Strongly exfoliated schist (graphite schist)	2.00
S.D.	1.35

Table 12.

Interaction (equipment × steepest gradient)	w
1×0	− 6.65
1×1	− 6.65
2×1	− 7.68
2×2	− 5.35
2×3	−10.30
2×4	− 5.75
2×5	− 6.19
2×6	− 5.49
2×7	− 7.02
2×L	− 6.33
3×0	−12.18
3×1	−13.48
3×2	−11.13
3×3	− 8.89
3×4	−12.94
3×5	−12.31
3×6	− 8.77
3×7	− 7.51
3×8	−13.49
3×L	−13.04
4×0	− 7.27
4×1	0.10
4×2	− 3.85
4×3	− 4.14
S.D.	3.66

Table 13.

Actual d.o.r. Estimated d.o.r.*	0	$\bar{0}$~	1~	2~	4~	6~	8~	10~	Total
2~	1								1
4~	1								1
6~	46	4	3						53
8~	61	2	1						64
10~	25	2	5	2	1		2		37
12~	6				3			1	10
14~	4							2	6
16~	2							1	3
18~								2	2
Total	146	8	9	2	4	0	2	6	177

* Estimated values added by 8.34.

The results described above explain, so to speak, the effects of primary d.o.r. However, they will change according to the amount of rainfall. Hence, we studied next the d.o.r. in each class of rainfall (see Table 14). Unfortunately, due to the incompleteness of information on rainfalls, we could only get a rough estimate of the range of d.o.r.

Variation of the range of d.o.r. between rainfall classes gives a key for the criterion of the amount of rainfall to order patrol.

The following table shows the range of d.o.r. and classes 25~, 75~, 150~ mm can be taken as those showing discontinuous changes.

Table 14. Range of d.o.r. in each rainfall class.

Continuous rainfall (mm)		0~	1~	25~	50~	75~	100~	150~
Equipment+steepest gradient ($x_i+y_j+w_{ij}$)	min	1.12	0.45	3.00	1.56	39.4	28.7	185.8
	max	10.39	11.52	70.99	60.55	272.9	1866.6*	2630.3*
Nature of soil (z_k)		1.80	0.92	9.27	2.65	65.2	80.9	520.9
Geological feature (u_l)		4.87	2.43	12.86	9.96	107.3	1931.3*	475.0

* indicates the existence of a special small unit.

Now there is a criterion to order patrol, adopted by Japanese National Railways, which has 3 standards, for example, in Ōboke-district 100, 75 and 25 mm of "continuous rainfall" in a slightly different sense from the one defined above, are the 1st, 2nd and 3rd levels for patrol order, respectively. The above-mentioned result seems to support the criterion. On the other hand the hourly rainfalls have certainly influence upon disaster. Therefore, it is desirable to make use of hourly rainfall for patrol order.

Analysis of hourly rainfall series in each rain gave us no forcasting formulae for total amount of continuous rainfall. Accordingly, we were obliged to use the regression formula of continuous rainfall to hourly rainfall. Thus we can use two sorts of criteria, that is, the one for hourly rainfall, and the other for continuous rainfall. When one of these two criteria is in force, the patrol is ordered.

An example in Ōboke-district is shown in the following table.

Elapsed hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Hourly rainfall	1	6	4	12	19	33*	35	21	26	35	22	22	21	14	4	5	4	1	2	5	4	8	3	4	4	5	5	4	7	7
Continuous rainfall	1	7	11	23	42	75	110**	131	157	192	214	236	257	271	275	280	284	285	287	292	296	304	307	311	315	320	325	329	336	343

* The patrol order level for hourly rainfall.

** The patrol order level for continuous rainfall.

4. An idea of optimal investment for natural disaster prevention

Japanese National Railways is one of three public corporations in Japan and their whole profit must be invested in new railway lines and other urgent programs.

Under this condition we put

S_n: income in the nth year
x_n: business expense in the nth year
y_n: investment for disaster prevention
z_n: total investment.

Then from past data we obtain the regression equation

$$(1) \qquad S_n = ax_n + bz_{n-1} + c\,, \qquad a,\, b,\, c:\ \text{constants}$$

and the profit (i.e. total investment) is given by

$$(2) \qquad z_n = S_n - x_n - y_n\,.$$

Let the discount coefficient be $\alpha = 1/(1+i)$, where i is the interest rate. Then the discount value of total investment during N years is

$$(3) \quad \sum_{n=1}^{N} z_n\alpha^n = \sum_{n=1}^{N}\{(a-1)(x_n + bx_{n-1} + \cdots + b^{n-1}x_1) + b^n z_0 + c(1 + b + \cdots + b^{n-1}) \\ -(y_n + by_{n-1} + \cdots + b^{n-1}y_1)\}\,\alpha^n\,.$$

If we can decide x_n as exogenous variables from the requested transportation volume, we have only to minimize

$$(4) \qquad f = \sum_{n=1}^{N}\alpha^n(y_n + by_{n-1} + \cdots + b^{n-1}y_1)$$

to maximize the discount value of investment (3).

Now let us assume the investment for disaster prevention divided into two parts: the one is the repair cost and the other is the investment for preventive works. Moreover, we assume that the repair cost diminishes oppositely to the increasing cumulative investment for preventive works:

$$I_n = I_0\Big(1 - \sum_{k=1}^{n-1} I_k^*/A\Big)$$

where I_n : repair cost in the nth year
I_0 : repair cost in the 1st year
I_k^* : investment for preventive works in the kth year
A : total investment for preventive works necessary to get to the situation of no damage.

Thus we get

$$(5) \qquad f=\frac{\alpha}{b\alpha-1}\left[I_0\left\{\alpha^N\frac{b^{N+1}-1}{b-1}-\frac{1-\alpha^{N+1}}{1-\alpha}\right\}+\sum_{n=1}^{N}I_n^*\left\{(b\alpha)^N\left(1-\frac{I_0}{A(b-1)}\right)b^{-n}\right.\right.$$
$$\left.\left.-\left(1-\frac{I_0\alpha}{A(1-\alpha)}\right)\alpha^{n-1}+\frac{I_0\alpha^N}{A}\left(1+\frac{1}{b-1}-\frac{1}{1-\alpha}\right)\right\}\right].$$

For 1961 to 1969 (1st and 2nd 5 year programs) we get

$$S_n=1.65(x_n+z_{n-1})-3100 \qquad \text{(100 million yen)}.$$

If we are allowed to assume $b=1.65$, putting particularly for Dosan-line $\alpha=0.934$ $(i=0.07)$, $I_0=1.39$ and $A=10$ (or 100), we obtain $I_N^*=10$ (or 100) because of the decreasing coefficient of I_n^*.

This result indicates that the later the investment for disaster prevention is, the smaller the f value is. In the actual situation, however, we shall have to consider conditions such as

$$(6) \qquad z_n\leqq kS_n$$

which is concerned with the regulation of investment, and

$$(7) \qquad 1-\sum_{n=1}^{N}I_n^*/A\leqq p$$

which is concerned with the disaster prevention effect.

Under these conditions we can get the solution by the linear programming method. When the investment is limited to disaster prevention, the case will be $b=0$. Then we have $I_1^*=10$ for $A=10$ but $I_N^*=100$ for $A=100$. This shows that the earlier the investment for disaster prevention is, the smaller the f value is, if A is so small that the inequality $1-\dfrac{I_0\alpha}{A(1-\alpha)}<0$ holds, and vice versa.

THE INSTITUTE OF STATISTICAL MATHEMATICS