NOTE ON OPTIMAL MACHINE SETTING

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When manufacturing products of a given standard by a given machine which has low precision in comparison with the standard, we usually get comparatively many products which are out of the standard. In such a case, it is desirable to get an optimal machine setting, taking into account a risk function. The purpose of this note is to treat this problem.

A machine is considered to have its own distribution of products when it is operated under a certain condition by an operator of a certain skill. Now, suppose this distribution be normal with mean μ and variance 1. The probability that we get products below the lower limit of the standard, or ones over the upper limit, varies according to the value which we can determine at the machine setting.

Let x, $M-B$ and $M+B$ be the attribute of the product, the lower and upper limits of the standard, respectively. The value of M can be assumed to be zero without loss of generality. Then $-B$ and B are the limits of a standard.

Now, assume the products are dealt with as follows.

1) The products below the lower limit $-B$ are regarded as scraps. In this case we suffer the loss K_s per a product.
2) The products over the upper limit of standard are reworked. The cost of rework is $K_R+k(x-B)^\alpha$ per a product, where K_R, k and α are given positive constants.

A treatment like this has a meaning only under the assumption that $K_R<K_S$. From 1) and 2) the cost function $c(x)$ is given as follows:

$$c(x)=\begin{cases} K_R+k\,(x-B)^\alpha & : \quad x\geqq B \\ 0 & : \quad B>x>-B \\ K_S & : \quad -B\geqq x \end{cases}$$

Then the average risk function $r(\mu)$ is represented as the function of machine setting μ.

$$(1) \qquad r(\mu)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} c(x)e^{-(x-\mu)^2/2}\,dx$$

Let δ be the Dirac's δ, that is.

$$\delta(x)=\begin{cases}1 & x\geqq 0\\ 0 & x<0\end{cases}, \qquad D(x)=\frac{d}{dx}\delta(x)=\begin{cases}1 & x=0\\ 0 & x\neq 0\end{cases}$$

Then we have

(2) $$c(x)=\delta(x-B)\{K_R+k(x-B)^\alpha\}+K_S(1-\delta(x+B))$$

Therefore

(3)

$$r(\mu)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Big[\delta(x-B)\{K_R+k(x-B)^\alpha\}+K_S(1-\delta(x+B))\Big]e^{-(x-\mu)^2/2}\,dx$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Big[\delta(t+\mu-B)\{K_R+k(t+\mu-B)^\alpha+K_S(1-\delta(x+B))\Big]e^{-t^2/2}\,dt$$

where $t=x-\mu$. For our purpose we decide machine setting μ such that it makes $r(\mu)$ minimum. Differentiating $r(\mu)$ by μ, we get

$$\frac{dr(\mu)}{d\mu}=\frac{1}{\sqrt{2\pi}}\Big[D(t+B-\mu)\{K_R+k(t+\mu-B)^\alpha\}\,e^{-t^2/2}$$

$$+k\alpha\int_{-\infty}^{\infty}\delta(t+\mu-B)(t+\mu-B)^{\alpha-1}e^{-t^2/2}\,dt$$

$$-K_S D(t+\mu-B)\,e^{-t^2/2}\Big]$$

Putting $\frac{dr(\mu)}{d\mu}=0$, $\frac{K_R}{K_S}=r$, and $\frac{k}{K_S}=k_0$, we obtain

(4) $$re^{-(B-\mu)^2/2}-e^{-(B+\mu)^2/2}+k_0\alpha\int_{B-\mu}^{\infty}(t+\mu-B)^{\alpha-1}e^{-t^2/2}dt=0\,.$$

Especially when $\alpha=0$, by solving (4) we get

(5) $$\mu=-\frac{1}{2B}\log r$$

When $\alpha=1$, 2, when an optimal machine setting by solving

(6) $$re^{-(B-\mu)^2/2}-e^{-(B+\mu)^2/2}+k_0\int_{B-\mu}^{\infty}e^{-t^2/2}\,dt=0$$

(7) $$(r+2k_0)e^{-(B-\mu)^2/2}-e^{-(B+\mu)^2/2}2k_0(\mu-B)\sqrt{2\pi}\Big(1-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{B-\mu}e^{-t^2/2}dt\Big)=0$$

In these cases, we can obtain the values of optimal machine setting μ_0 for various of B, k_0 and r by drawing monographs of each equations.

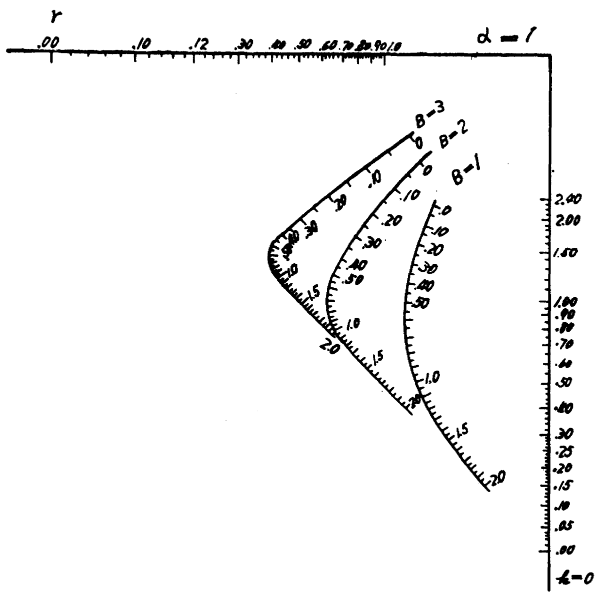

Fig. 1. Monograph of equation (6) for $B=1, 2, 3$

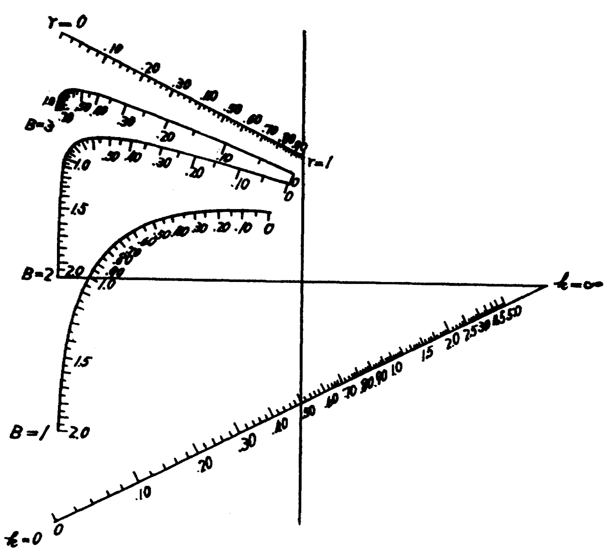

Fig. 2. Monograph of equation (7) for $B=1, 2, 3$

For example, from Fig. 1 we get $\mu_0=0.8$ for $\alpha=1$, $r=0.10$, and $k_0=0.10$. Therefore the optimal machine setting is to set production machine so that the mean value of distribution of its products may be $M+0.8\sigma$.

Tables 1, 2 indicate the values of optimal machine setting μ_0 obtained from Fig. 1 and Fig. 2 for various of r and k_0.

Table 1. The values of optimal machine setting for various values of r and k_0, when $\alpha=2$ and $B=1$.

k_0 \ r	0.02	0.04	0.0	0.08	0.10	0.20	0.30	0.40	0.50	0.60	0.70
0.02	1.30	1.22	1.14	1.07	1.0	0.74	0.57	0.44	0.33	0.23	0.18
0.04	1.12	1.05	1.00	0.94	0.89	0.69	0.54	0.42	0.32	0.23	0.17
0.06	1.00	0.94	0.90	0.86	0.82	0.64	0.52	0.39	0.30	0.22	0.15
0.08	0.92	0.88	0.84	0.81	0.77	0.61	0.48	0.38	0.28	0.21	0.14
0.10	0.85	0.78	0.78	0.75	0.72	0.58	0.47	0.51	0.27	0.20	0.13

Table 2. The values of optimal machine setting for various values of r and k_0, when $\alpha=1$ and $B=2$.

k_0 \ r	0.02	0.04	0.08	0.20	0.30	0.40	0.50	0.60	0.70
0.05	0.78	0.70	0.58	0.38	0.29	0.22	0.17	0.12	0.08
0.10	0.67	0.60	0.52	0.35	0.27	0.21	0.16	0.11	0.08
0.20	0.53	0.49	0.44	0.32	0.24	0.18	0.14	0.09	0.06
0.40	0.39	0.37	0.33	0.25	0.19	0.14	0.10	0.07	0.04
0.60	0.31	0.29	0.27	0.19	0.15	0.11	0.07	0.04	0.02
0.80	0.25	0.23	0.20	0.15	0.11	0.07	0.04	0.02	0.00

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