Central limit theorems for vector-valued composite functionals with smoothing and applications

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Abstract

This paper focuses on vector-valued composite functionals, which may be nonlinear in probability. Our goal is establishing central limit theorems for these functionals when employed by mixed estimators. Our study is relevant to the evaluation and comparison of risk in decision-making contexts and extends to functionals that arise in machine learning. A generalized family of composite risk functionals is presented, which encompasses coherent risk measures, including systemic risk. The paper makes two main contributions. First, we analyze vector-valued functionals and provide a framework for evaluating high-dimensional risks. This enables comparison of multiple risk measures and supports estimation and asymptotic analysis of systemic risk and its optimal value in decision-making. Second, we derive new central limit theorems for optimized composite functionals using mixed estimators, including empirical and smoothed types. We give verifiable conditions for central limit formulae and demonstrate their applicability to several risk measures.

Keywords Coherent measure of risk · Stochastic programming · Systemic risk

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