Posterior contraction rate and asymptotic Bayes optimality for one group global–local shrinkage priors in sparse normal means problem

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Abstract

We study inference on the mean vector of the normal means model in sparse asymptotic settings when it is modelled by broad classes of one-group global–local continuous shrinkage priors. We prove that the resulting posterior distributions contract around the truth at a near minimax rate with respect to squared L_2 loss when the global shrinkage parameter is estimated in empirical Bayesian ways or arbitrary priors supported on some appropriate interval are assigned to it. We then employ an intuitive multiple testing rule (using full Bayes treatment with global–local priors) in a problem of simultaneous testing (with additive misclassification loss) for the components of the mean assuming they are iid from a two-groups prior. In a first result of its kind, risk of our testing rule is shown to asymptotically match (up to a constant) that of the optimal rule in the two-groups setting.

Keywords Asymptotic minimaxity · Near-minimax rate · One-group prior · Sparsity · ABOS

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