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A distance covariance test of independence in high dimension, low sample size contexts

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Abstract
To check the mutual independence of a high-dimensional random vector without Gaussian assumption, Yao et al. (Journal of the Royal Statistical Society Series B, 80,455–480, 2018) recently introduced an important test by virtue of pairwise distance covariances. Despite its usefulness, the state-of-art test tends to have unsatisfactory size performance when the sample size is small. The present paper provides a theoretical explanation about this phenomenon, and accordingly proposes a new test in high dimension, low sample size contexts. The new test can be even justified as the dimension tends to infinity, regardless of whether the sample size is fixed or diverges. The power of the proposed distance covariance test is also investigated. To examine our theoretical findings and check the performance of the new test, simulation studies are applied. We further illustrate the proposed method by empirical analysis of a real dataset.

Keywords Distance covariance · High dimension · Low sample size · Independence test

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