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SPECIAL PAPER (WITH DISCUSSION)

Mode-based estimation of the center of symmetry

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Abstract

In the mean-median-mode triad of univariate centrality measures, the mode has been overlooked for estimating the center of symmetry in continuous and unimodal settings. This paper expands on the connection between kernel mode estimators and M-estimators for location, bridging the gap between the nonparametrics and robust statistics communities. The variance of modal estimators is studied in terms of a bandwidth parameter, establishing conditions for an optimal solution that outperforms the household sample mean. A purely nonparametric approach is adopted, modeling heavy-tailedness through regular variation. The results lead to an estimator proposal that includes a novel one-parameter family of kernels with compact support, offering extra robustness and efficiency. The effectiveness and versatility of the new method are demonstrated in a real-world case study and a thorough simulation study, comparing favorably to traditional and more competitive alternatives. Several myths about the mode are clarified along the way, reopening the quest for flexible and efficient nonparametric estimators.

Keywords Kernel mode estimator · Center of symmetry · Unimodality · Regularly varying density · Redescending M-estimator · Efficient nonparametric estimation

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