



Uniformly consistent proportion estimation for composite hypotheses via integral equations: “the case of Gamma random variables”

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Abstract

We consider estimating the proportion of random variables for two types of composite null hypotheses: (i) the means of the random variables belonging to a non-empty, bounded interval; (ii) the means of the random variables belonging to an unbounded interval that is not the whole real line. For each type of composite null hypotheses, uniformly consistent estimators of the proportion of false null hypotheses are constructed for random variables whose distributions are members of the Gamma family. Further, uniformly consistent estimators of certain functions of a bounded null on the means are provided for the random variables mentioned earlier. These functions are continuous and of bounded variation. The estimators are constructed via solutions to Lebesgue-Stieltjes integral equations and harmonic analysis, do not rely on a concept of p -value, and have various applications.

Keywords Composite null hypothesis · Harmonic analysis · Lebesgue-Stieltjes integral equations · Proportion of false null hypotheses

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