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On uniform consistency of nonparametric estimators smoothed by the gamma kernel

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Abstract

This paper documents a set of uniform consistency results with rates for nonparametric density and regression estimators smoothed by the gamma kernel having support on the nonnegative real line. It is known that this kernel can well calibrate the shapes of 'cost' distributions that are characterized by a sharp peak in the vicinity of the origin and a long right tail. In this paper, weak and strong uniform consistency and corresponding convergence rates of gamma kernel estimators are explored in a multivariate framework. Our analysis is built on compact sets expanding to the nonnegative orthant and general sequences of smoothing parameters. The results are useful for asymptotic analysis of two-step semiparametric estimation using a first-step kernel estimate as a plug-in.

Keywords Boundary bias · Density derivative estimation · Density estimation · Gamma kernel · Nonparametric regression estimation · Uniform convergence

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