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Non-explicit formula of boundary crossing probabilities by the Girsanov theorem

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Abstract

This paper derives several formulae for the probability that a Wiener process, which has a stochastic drift and random variance, crosses a one-sided stochastic boundary within a finite time interval. A non-explicit formula is first obtained by the Girsanov theorem when considering an equivalent probability measure in which the boundary is constant and equal to its starting value. A more explicit formula is then achieved by decomposing the Radon–Nikodym derivative inverse. This decomposition expresses it as the product of a random variable, which is measurable with respect to the Wiener process's final value, and an independent random variable. We also provide an explicit formula based on a strong theoretical assumption. To apply the Girsanov theorem, we assume that the difference between the drift increment and the boundary increment, divided by the standard deviation, is absolutely continuous. Additionally, we assume that its derivative satisfies Novikov's condition.

Keywords Mathematical statistics · Sequential analysis · First-passage time problem · Boundary crossing probabilities · Stochastic boundary process · Wiener process · Girsanov theorem

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