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Penalized estimation for non-identifiable models

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Abstract

We derive asymptotic properties of penalized estimators for singular models for which identifiability may break and the true parameter values can lie on the boundary of the parameter space. Selection consistency of the estimators is also validated. The problem that the true values lie on the boundary is solved by our previous results applicable to singular models, besides, penalized estimation and non-ergodic statistics. To overcome non-identifiability, we consider a suitable penalty such as the non-convex Bridge and the adaptive Lasso that stabilize the asymptotic behavior of the estimator and shrink inactive parameters. Then the estimator converges to one of the most parsimonious values among all the true values. The oracle property can also be obtained even if likelihood ratio tests for model selection are labor intensive due to singularity of models. Examples are: a superposition of parametric proportional hazard models and a counting process having intensity with multicollinear covariates.

Keywords Quasi-likelihood · Penalized likelihood · Boundary · Non-identifiability · Variable selection · Superposed process · Multicollinearity · Proportional hazard model

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