# A delineation of new classes of exponential dispersion models supported on the set of nonnegative integers

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## Abstract

The aim of this paper is to delineate a set of new classes of natural exponential families and their associated exponential dispersion models whose probability distributions are supported on the set of nonnegative integers with positive mass on 0 and 1. The new classes are obtained by considering a specific form of their variance functions. We show that the distributions of all these classes are supported on nonnegative integers, that they are infinitely divisible, and that they are skewed to the right, leptokurtic, over-dispersed, and zero-inflated (relative to the Poisson class). Accordingly, these new classes significantly enrich the set of probability models for modeling zero-inflated and over-dispersed count data. Furthermore, we elaborate on numerical techniques how to compute the distributions of our classes, and apply these to an actual data experiment.

**Keywords** Discrete distribution · Exponential dispersion model · Infinitely divisible · Lagrange inversion formula · Natural exponential family · Numerical computations · Variance function

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