Annals of the Institute of Statistical Mathematics (2024) 76:263–287
https://doi.org/10.1007/s10463-023-00883-5

Multivariate frequency polygon for stationary random fields

Michel Carbon[1] · **Thierry Duchesne**[1]

Received: 9 September 2022 / Revised: 3 August 2023 / Accepted: 29 August 2023 /
Published online: 8 November 2023

Abstract

The purpose of this paper is to investigate the multivariate frequency polygon as a density estimator for stationary random fields indexed by multidimensional lattice points space. Optimal cell widths that asymptotically minimize integrated mean square error (IMSE) are derived. Under weak conditions, the IMSE of frequency polygons achieves the same rate of convergence to zero as that of kernel estimators. The frequency polygon can also attain the optimal uniform rate of convergence and the almost sure convergence under general conditions. Finally, a result of L^1 convergence is given. Frequency polygons thus appear to be very good density estimators with respect to the criteria of IMSE, of uniform convergence, of almost sure convergence and of L^1 convergence. We apply our results to simulated data and real data.

Keywords Bandwidth · Density estimation · Frequency polygons · Mixing field · Random field

✉ Thierry Duchesne
thierry.duchesne@mat.ulaval.ca

[1] Department of Mathematics and Statistics, Université Laval, 1056, avenue de la Médecine, Québec, QC G1V 0A6, Canada