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Approximating symmetrized estimators of scatter via balanced incomplete *U*-statistics

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Abstract

We derive limiting distributions of symmetrized estimators of scatter. Instead of considering all $n(n-1)/2$ pairs of the n observations, we only use nd suitably chosen pairs, where $d \geq 1$ is substantially smaller than n. It turns out that the resulting estimators are asymptotically equivalent to the original one whenever $d = d(n) \to \infty$ at arbitrarily slow speed. We also investigate the asymptotic properties for arbitrary fixed d. These considerations and numerical examples indicate that for practical purposes, moderate fixed values of d between 10 and 20 yield already estimators which are computationally feasible and rather close to the original ones.

Keywords Asymptotic normality · Incomplete U-statistic · Independent component analysis · Linear expansion · U-statistic

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