



# Asymptotic theory in network models with covariates and a growing number of node parameters

Qiuping Wang<sup>1</sup> · Yuan Zhang<sup>2</sup> · Ting Yan<sup>3</sup>

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## Abstract

We propose a general model that jointly characterizes degree heterogeneity and homophily in weighted, undirected networks. We present a moment estimation method using node degrees and homophily statistics. We establish consistency and asymptotic normality of our estimator using novel analysis. We apply our general framework to three applications, including both exponential family and non-exponential family models. Comprehensive numerical studies and a data example also demonstrate the usefulness of our method.

**Keywords**  $\beta$ -Model · Degree heterogeneity · Network homophily · Network method of moments

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Qiuping Wang and Yuan Zhang have contributed equally to this work.

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✉ Ting Yan  
tingyanty@mail.ccnu.edu.cn

Qiuping Wang  
qp.wang@mails.ccnu.edu.cn

Yuan Zhang  
yzhanghf@stat.osu.edu

<sup>1</sup> School of Mathematics and Statistics, Zhaoqing University, Zhaoqing Avenue, Duanzhou District, Zhaoqing 526061, Guangdong, China

<sup>2</sup> Department of Statistics, The Ohio State University, 1958 Neil Avenue, Columbus, OH 43210, USA

<sup>3</sup> Department of Statistics, Central China Normal University, 152 Luoyu Road, Wuhan 430079, Hubei, China