On the choice of the optimal single order statistic in quantile estimation

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Abstract

We study the classical statistical problem of the estimation of quantiles by order statistics of the random sample. For fixed sample size, we determine the single order statistic which is the optimal estimator of a quantile of given order. We propose a totally new approach to the problem, since our optimality criterion is based on the use of nonparametric sharp upper and lower bounds on the bias of the estimation. First, we determine the explicit analytic expressions for the bounds, and then, we choose the order statistic for which the upper and lower bound are simultaneously as close to 0 as possible. The paper contains rigorously proved theoretical results which can be easily implemented in practise. This is also illustrated with numerical examples.

Keywords Bias · Nonparametric statistics · Order statistics · Quantile estimation · Sharp bounds · Small sample

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