



# Inference using an exact distribution of test statistic for random-effects meta-analysis

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## Abstract

Random-effects meta-analysis serves to integrate the results of multiple studies with methods such as moment estimation and likelihood estimation duly proposed. These existing methods are based on asymptotic normality with respect to the number of studies. However, the test and interval estimation deviate from the nominal significance level when integrating a small number of studies. Although a method for constructing more conservative intervals has been recently proposed, the exact distribution of test statistic for the overall treatment effect is not well known. In this paper, we provide an almost-exact distribution of the test statistic in random-effects meta-analysis and propose the test and interval estimation using the almost-exact distribution. Simulations demonstrate the accuracy of estimation and application to existing meta-analysis using the method proposed here. With known variance parameters, the estimation performance using the almost-exact distribution always achieves the nominal significance level regardless of the number of studies and heterogeneity. We also propose some methods to construct a conservative interval estimation, even when the variance parameters are unknown, and present their performances via simulation and an application to Alzheimer's disease meta-analysis.

**Keywords** Exact distribution · Meta-analysis · Random-effects model · Test statistic

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