Asymptotic equivalence for nonparametric regression with dependent errors: Gauss–Markov processes

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Abstract

For the class of Gauss–Markov processes we study the problem of asymptotic equivalence of the nonparametric regression model with errors given by the increments of the process and the continuous time model, where a whole path of a sum of a deterministic signal and the Gauss–Markov process can be observed. We derive sufficient conditions which imply asymptotic equivalence of the two models. We verify these conditions for the special cases of Sobolev ellipsoids and Hölder classes with smoothness index $> 1/2$ under mild assumptions on the Gauss–Markov process. To give a counterexample, we show that asymptotic equivalence fails to hold for the special case of Brownian bridge. Our findings demonstrate that the well-known asymptotic equivalence of the Gaussian white noise model and the nonparametric regression model with i.i.d. standard normal errors (see Brown and Low (Ann Stat 24:2384–2398, 1996)) can be extended to a setup with general Gauss–Markov noises.

Keywords Asymptotic equivalence · Nonparametric regression · Dependent errors · Gauss–Markov process · Triangular kernel

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