



# On the rate of convergence of image classifiers based on convolutional neural networks

Michael Kohler<sup>1</sup> · Adam Krzyżak<sup>2</sup> · Benjamin Walter<sup>1</sup>

Received: 20 May 2021 / Revised: 16 March 2022 / Accepted: 17 March 2022 /

Published online: 27 April 2022

© The Institute of Statistical Mathematics, Tokyo 2022

## Abstract

Image classifiers based on convolutional neural networks are defined, and the rate of convergence of the misclassification risk of the estimates towards the optimal misclassification risk is analyzed. Under suitable assumptions on the smoothness and structure of a posteriori probability, the rate of convergence is shown which is independent of the dimension of the image. This proves that in image classification, it is possible to circumvent the curse of dimensionality by convolutional neural networks. Furthermore, the obtained result gives an indication why convolutional neural networks are able to outperform the standard feedforward neural networks in image classification. Our classifiers are compared with various other classification methods using simulated data. Furthermore, the performance of our estimates is also tested on real images.

**Keywords** Curse of dimensionality · Convolutional neural networks · Image classification · Rate of convergence

---

Michael Kohler was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Projektnummer 449102119.

Adam Krzyżak gratefully acknowledges the support from the Natural Sciences and Engineering Research Council of Canada Under Grant RGPIW-2015-06412.

---

✉ Benjamin Walter  
bwalter@mathematik.tu-darmstadt.de

Michael Kohler  
kohler@mathematik.tu-darmstadt.de

Adam Krzyżak  
krzyzak@cs.concordia.ca

<sup>1</sup> Fachbereich Mathematik, Technische Universität Darmstadt, Schlossgartenstr. 7, 64289 Darmstadt, Germany

<sup>2</sup> Department of Computer Science and Software Engineering, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, QC H3G 1M8, Canada