



# Outcome regression-based estimation of conditional average treatment effect

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## Abstract

The research is about a systematic investigation on the following issues. First, we construct different outcome regression-based estimators for conditional average treatment effect under, respectively, true, parametric, nonparametric and semiparametric dimension reduction structure. Second, according to the corresponding asymptotic variance functions when supposing the models are correctly specified, we answer the following questions: what is the asymptotic efficiency ranking about the four estimators in general? how is the efficiency related to the affiliation of the given covariates in the set of arguments of the regression functions? what do the roles of bandwidth and kernel function selections play for the estimation efficiency; and in which scenarios should the estimator under semiparametric dimension reduction regression structure be used in practice? Meanwhile, the results show that any outcome regression-based estimation should be asymptotically more efficient than any inverse probability weighting-based estimation. Several simulation studies are conducted to examine the finite sample performances of these estimators, and a real dataset is analyzed for illustration.

**Keywords** Asymptotic variance · Conditional average treatment effect · Regression causal effect · Sufficient dimension reduction

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