

## Directed hybrid random networks mixing preferential attachment with uniform attachment mechanisms

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## Abstract

Motivated by the complexity of network data, we propose a directed hybrid random network that mixes preferential attachment (PA) rules with uniform attachment rules. When a new edge is created, with probability  $p \in (0, 1)$ , it follows the PA rule. Otherwise, this new edge is added between two uniformly chosen nodes. Such mixture makes the in- and out-degrees of a fixed node grow at a slower rate, compared to the pure PA case, thus leading to lighter distributional tails. For estimation and inference, we develop two numerical methods which are applied to both synthetic and real network data. We see that with extra flexibility given by the parameter p, the hybrid random network provides a better fit to real-world scenarios, where lighter tails from in- and out-degrees are observed.

**Keywords** Preferential attachment  $\cdot$  Uniform attachment  $\cdot$  In- and out-degrees  $\cdot$  Power laws  $\cdot$  Random networks

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