



# Bahadur efficiency of the maximum likelihood estimator and one-step estimator for quasi-arithmetic means of the Cauchy distribution

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## Abstract

Some quasi-arithmetic means of random variables easily give unbiased strongly consistent closed-form estimators of the joint of the location and scale parameters of the Cauchy distribution. The one-step estimators of those quasi-arithmetic means of the Cauchy distribution are considered. We establish the Bahadur efficiency of the maximum likelihood estimator and the one-step estimators. We also show that the rate of the convergence of the mean-squared errors achieves the Cramér–Rao bound. Our results are also applicable to the circular Cauchy distribution .

**Keywords** Bahadur efficiency · Cauchy distribution · Maximum likelihood estimator · One-step estimator

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