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Bayes factor asymptotics for variable selection in the Gaussian process framework

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Abstract

We investigate Bayesian variable selection in models driven by Gaussian processes, which allows us to treat linear, nonlinear and nonparametric models, in conjunction with even dependent setups, in the same vein. We consider the Bayes factor route to variable selection, and develop a general asymptotic theory for the Gaussian process framework in the "large p, large n" settings even with $p \gg n$, establishing almost sure exponential convergence of the Bayes factor under appropriately mild conditions. The fixed p setup is included as a special case. To illustrate, we apply our result to variable selection in linear regression, Gaussian process model with squared exponential covariance function accommodating the covariates, and a first-order autoregressive process with time-varying covariates. We also follow up our theoretical investigations with ample simulation experiments in the above regression contexts and variable selection in a real, riboflavin data consisting of 71 observations and 4088 covariates. For implementation of variable selection using Bayes factors, we develop a novel and effective general-purpose transdimensional, transformation-based Markov chain Monte Carlo algorithm, which has played a crucial role in simulated and real data applications.

Keywords Strong consistency · Kullback–Leibler divergence · Integrated Bayes factor · Squared exponential kernel · MCMC · Variable selection

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