



Characterizing the optimal solutions to the isotonic regression problem for identifiable functionals

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Abstract

In general, the solution to a regression problem is the minimizer of a given loss criterion and depends on the specified loss function. The nonparametric isotonic regression problem is special, in that optimal solutions can be found by solely specifying a functional. These solutions will then be minimizers under all loss functions simultaneously as long as the loss functions have the requested functional as the Bayes act. For the functional, the only requirement is that it can be defined via an identification function, with examples including the expectation, quantile, and expectile functionals. Generalizing classical results, we characterize the optimal solutions to the isotonic regression problem for identifiable functionals by rigorously treating these functionals as set-valued. The results hold in the case of totally or partially ordered explanatory variables. For total orders, we show that any solution resulting from the pool-adjacent-violators algorithm is optimal.

Keywords Order-restricted optimization problems · Partial order · Simultaneous optimality · Pool-adjacent-violators algorithm · Consistent loss functions

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