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Characterizations of the normal distribution via the independence of the sample mean and the feasible definite statistics with ordered arguments

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Abstract
It is well known that the independence of the sample mean and the sample variance characterizes the normal distribution. By using Anosov's theorem, we further investigate the analogous characteristic properties in terms of the sample mean and some feasible definite statistics. The latter statistics introduced in this paper for the first time are based on nonnegative, definite and continuous functions of ordered arguments with positive degree of homogeneity. The proposed approach seems to be natural and can be used to derive easily characterization results for many feasible definite statistics, such as known characterizations involving the sample variance, sample range as well as Gini's mean difference.

Keywords Characterization of distributions · Order statistics · Anosov's theorem · Benedetti's inequality · Sample mean · Sample variance · Sample range · Gini's mean difference

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