Asymptotic linear expansion of regularized M-estimators

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Abstract
Parametric high-dimensional regression requires regularization terms to get inter-
pretable models. The respective estimators correspond to regularized M-functionals
which are naturally highly nonlinear. Their Gâteaux derivative, i.e., their influence
curve linearizes the asymptotic bias of the estimator, but only up to a remainder
term which is not guaranteed to tend (sufficiently fast) to zero uniformly on suitable
tangent sets without profound arguments. We fill this gap by studying, in a unified
framework, under which conditions the M-functionals corresponding to convex pen-
alties as regularization are compactly differentiable, so that the estimators admit an
asymptotically linear expansion. This key ingredient allows influence curves to rea-
sonably enter model diagnosis and enable a fast, valid update formula, just requiring
an evaluation of the corresponding influence curve at new data points. Moreover,
this paves the way for optimally-robust estimators, bounding the influence curves in
a suitable way.

Keywords Asymptotic linear expansion · Regularized M-estimators · Influence
curves

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