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Efficient estimation methods for non-Gaussian regression models in continuous time

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Abstract
In this paper, we develop an efficient nonparametric estimation theory for continuous time regression models with non-Gaussian Lévy noises in the case when the unknown functions belong to Sobolev ellipses. Using the Pinsker's approach, we provide a sharp lower bound for the normalized asymptotic mean square accuracy. However, the main result obtained by Pinsker for the Gaussian white noise model is not correct without additional conditions for the ellipse coefficients. We find such constructive sufficient conditions under which we develop efficient estimation methods. We show that the obtained conditions hold for the ellipse coefficients of an exponential form. For exponential coefficients, the sharp lower bound is calculated in explicit form. Finally, we apply this result to signals number detection problems in multi-pass connection channels and we obtain an almost parametric convergence rate that is natural for this case, which significantly improves the rate with respect to power-form coefficients.

Keywords Regression model · Lévy process · Asymptotic efficiency · Weighted least squares estimates · Pinsker constant · Quadratic risk

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