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Wasserstein statistics in one-dimensional location scale models

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Abstract

Wasserstein geometry and information geometry are two important structures to be introduced in a manifold of probability distributions. Wasserstein geometry is defined by using the transportation cost between two distributions, so it reflects the metric of the base manifold on which the distributions are defined. Information geometry is defined to be invariant under reversible transformations of the base space. Both have their own merits for applications. In this study, we analyze statistical inference based on the Wasserstein geometry in the case that the base space is one-dimensional. By using the location-scale model, we further derive the W-estimator that explicitly minimizes the transportation cost from the empirical distribution to a statistical model and study its asymptotic behaviors. We show that the W-estimator is consistent and explicitly give its asymptotic distribution by using the functional delta method. The W-estimator is Fisher efficient in the Gaussian case.

Keywords Information geometry · Location-scale model · Optimal transport · Wasserstein distance

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