Annals of the Institute of Statistical Mathematics (2021) 73:1187–1202
https://doi.org/10.1007/s10463-020-00779-8

Improper versus finitely additive distributions as limits of countably additive probabilities

Pierre Druilhet[1] · **Erwan Saint Loubert Bié**[1]

Received: 27 August 2019 / Revised: 9 October 2020 / Accepted: 10 November 2020 /
Published online: 3 January 2021
© The Institute of Statistical Mathematics, Tokyo 2021

Abstract

The Bayesian paradigm with proper priors can be extended either to improper distributions or to finitely additive probabilities (FAPs). Improper distributions and diffuse FAPs can be seen as limits of proper distribution sequences for specific convergence modes. In this paper, we compare these two kinds of limits. We show that improper distributions and FAPs represent two distinct features of the limit behavior of a sequence of proper distribution. More specifically, an improper distribution characterizes the behavior of the sequence inside the domain, whereas diffuse FAPs characterizes how the mass concentrates on the boundary of the domain. Therefore, a diffuse FAP cannot be seen as the counterpart of an improper distribution. As an illustration, we consider several approach to define uniform FAP distributions on natural numbers as an equivalent of improper flat prior. We also show that expected logarithmic convergence may depend on the chosen sequence of compact sets.

Keywords Bayesian statistics · Improper distribution · Finitely additive probability · Q-vague convergence · Uniform distribution · Expected logarithmic convergence. · Remote probability

✉ Pierre Druilhet
pierre.druilhet@uca.fr

[1] CNRS, LMBP, UMR 6620, Université Clermont Auvergne, Campus des Cézeaux, 3 Place Vasarely, TSA 60026, CS 60026, 63178 Aubiere Cedex, France