Asymptotic behavior of mean density estimators based on a single observation: the Boolean model case

Federico Camerlenghi[1,2,3] · Claudio Macci[4] · Elena Villa[5]

Abstract

The mean density estimation of a random closed set in $\mathbb{R}^d$, based on a single observation, is a crucial problem in several application areas. In the case of stationary random sets, a common practice to estimate the mean density is to take the n-dimensional volume fraction with observation window as large as possible. In the present paper, we provide large and moderate deviation results for these estimators when the random closed set Θ_n belongs to the quite general class of stationary Boolean models with Hausdorff dimension $n < d$. Moreover, we establish a central limit theorem and a Berry–Esseen bound for the family of estimators under study. Our findings allow to recover some well-known results in the literature on Boolean models. Finally, we also provide a guideline for the estimation of the mean density of nonstationary Boolean models characterized by high intensity of the underlying Poisson point process.

Keywords Hausdorff measure · Large deviations · Moderate deviations · Point processes · Stochastic geometry

✉ Federico Camerlenghi
federico.camerlenghi@unimib.it

Claudio Macci
macci@mat.uniroma2.it

Elena Villa
elena.villa@unimi.it

[1] Dipartimento di Economia, Metodi Quantitativi e Strategie di Impresa, Università di Milano - Bicocca, Piazza dell'Ateneo Nuovo 1, 20126 Milan, Italy

[2] Collegio Carlo Alberto, Turin, Italy

[3] BIDSA, Bocconi University, Milan, Italy

[4] Dipartimento di Matematica, Università di Roma Tor Vergata, Via della Ricerca Scientifica, 00133 Rome, Italy

[5] Dipartimento di Matematica, Università degli Studi di Milano, Via Saldini 50, 20133 Milan, Italy