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Mellin–Meijer kernel density estimation on $\mathbb{R}^+$

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Abstract

Kernel density estimation is a nonparametric procedure making use of the smoothing power of the convolution operation. Yet, it performs poorly when the density of a positive variable is estimated, due to boundary issues. So, various extensions of the kernel estimator allegedly suitable for $\mathbb{R}^+$-supported densities, such as those using asymmetric kernels, abound in the literature. Those, however, are not based on any valid smoothing operation. By contrast, in this paper a kernel density estimator is defined through the Mellin convolution, the natural analogue on $\mathbb{R}^+$ of the usual convolution. From there, a class of asymmetric kernels related to Meijer G-functions is suggested, and asymptotic properties of this 'Mellin–Meijer kernel density estimator' are presented. In particular, its pointwise- and L_2-consistency (with optimal rate of convergence) are established for a large class of densities, including densities unbounded at 0 and showing power-law decay in their right tail.

Keywords Kernel density estimator · Boundary issues · Asymmetric kernels · Mellin transform · Meijer G-functions

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