

## Identifying shifts between two regression curves

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## Abstract

This article studies the problem whether two convex (concave) regression functions modelling the relation between a response and covariate in two samples differ by a shift in the horizontal and/or vertical axis. We consider a nonparametric situation assuming only smoothness of the regression functions. A graphical tool based on the derivatives of the regression functions and their inverses is proposed to answer this question and studied in several examples. We also formalize this question in a corresponding hypothesis and develop a statistical test. The asymptotic properties of the corresponding test statistic are investigated under the null hypothesis and local alternatives. In contrast to most of the literature on comparing shape invariant models, which requires independent data the procedure is applicable for dependent and non-stationary data. We also illustrate the finite sample properties of the new test by means of a small simulation study and two real data examples.

**Keywords** Comparison of curves · Nonparametric regression · Hypothesis testing

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