



# Gaussian graphical models with toric vanishing ideals

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## Abstract

Gaussian graphical models are semi-algebraic subsets of the cone of positive definite covariance matrices. They are widely used throughout natural sciences, computational biology and many other fields. Computing the vanishing ideal of the model gives us an implicit description of the model. In this paper, we resolve two conjectures given by Sturmfels and Uhler. In particular, we characterize those graphs for which the vanishing ideal of the Gaussian graphical model is generated in degree 1 and 2. These turn out to be the Gaussian graphical models whose ideals are toric ideals, and the resulting graphs are the 1-clique sums of complete graphs.

**Keywords** Clique sum · Toric ideals · SAGBI bases · Initial algebra

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