



Hypothesis tests for high-dimensional covariance structures

Aki Ishii¹ · Kazuyoshi Yata² · Makoto Aoshima²

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Abstract

We consider hypothesis testing for high-dimensional covariance structures in which the covariance matrix is a (i) scaled identity matrix, (ii) diagonal matrix, or (iii) intraclass covariance matrix. Our purpose is to systematically establish a nonparametric approach for testing the high-dimensional covariance structures (i)–(iii). We produce a new common test statistic for each covariance structure and show that the test statistic is an unbiased estimator of its corresponding test parameter. We prove that the test statistic establishes the asymptotic normality. We propose a new test procedure for (i)–(iii) and evaluate its asymptotic size and power theoretically when both the dimension and sample size increase. We investigate the performance of the proposed test procedure in simulations. As an application of testing the covariance structures, we give a test procedure to identify an eigenvector. Finally, we demonstrate the proposed test procedure by using a microarray data set.

Keywords Cross-data-matrix methodology · Diagonal structure · HDLSS · Intraclass correlation model · Test of eigenvector · Unbiased estimate

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✉ Makoto Aoshima
aoshima@math.tsukuba.ac.jp

Aki Ishii
a.ishii@rs.tus.ac.jp

Kazuyoshi Yata
yata@math.tsukuba.ac.jp

¹ Department of Information Sciences, Tokyo University of Science, 2641 Yamazaki, Noda-shi, Chiba 278-8510, Japan

² Institute of Mathematics, University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki 305-8571, Japan