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Multiresolution analysis of point processes and statistical thresholding for Haar wavelet-based intensity estimation

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Abstract

We take a wavelet-based approach to the analysis of point processes and the estimation of the first-order intensity under a continuous-time setting. A Haar wavelet multiresolution analysis is formulated which motivates the definition of homogeneity at different scales of resolution, termed J-th level homogeneity. Further to this, the activity in a point process' first-order behaviour at different scales of resolution is also defined and termed L-th level innovation. Likelihood ratio tests for both these properties are proposed with asymptotic distributions provided, even when only a single realization is observed. The test for L-th level innovation forms the basis for a collection of statistical strategies for thresholding coefficients in a wavelet-based estimator of the intensity function. These thresholding strategies outperform the existing local hard thresholding strategy on a range of simulation scenarios. This methodology is applied to NetFlow data, characterizing multiscale behaviour on computer networks.

Keywords Wavelets · Multiresolution analysis · Poisson process · Likelihood ratio test · Statistical thresholding

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