



Valid p -values and expectations of p -values revisited

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Abstract

We focus on valid definitions of p -values. A valid p -value (VpV) statistic can be used to make a prefixed level- α decision. In this context, Kolmogorov–Smirnov goodness-of-fit tests and the normal two-sample problem are considered. We examine an issue regarding the goodness-of-fit testability based on a single observation. We exemplify constructions of new test procedures, advocating practical reasons to implement VpV mechanisms. The VpV framework induces an extension of the conventional expected p -value (EPV) tool for measuring the performance of a test. Associating the EPV concept with the receiver operating characteristic (ROC) curve methodology, a well-established biostatistical approach, we propose a Youden’s index-based optimality to derive critical values of tests. In these terms, the significance level $\alpha = 0.05$ is suggested. We introduce partial EPV’s to characterize properties of tests including their unbiasedness. We provide the intrinsic relationship between the Bayes Factor (BF) test statistic and the BF of test statistics.

Keywords AUC · Bayes Factor · Kolmogorov–Smirnov tests · Likelihood ratio · p -value · ROC curve · Pooled data · Single observation · Type I error rate · Youden’s index

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