



Fixed point characterizations of continuous univariate probability distributions and their applications

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Abstract

By extrapolating the explicit formula of the zero-bias distribution occurring in the context of Stein’s method, we construct characterization identities for a large class of absolutely continuous univariate distributions. Instead of trying to derive characterizing distributional transformations that inherit certain structures for the use in further theoretic endeavors, we focus on explicit representations given through a formula for the density- or distribution function. The results we establish with this ambition feature immediate applications in the area of goodness-of-fit testing. We draw up a blueprint for the construction of tests of fit that include procedures for many distributions for which little (if any) practicable tests are known. To illustrate this last point, we construct a test for the Burr Type XII distribution for which, to our knowledge, not a single test is known aside from the classical universal procedures.

Keywords Burr Type XII distribution · Density approach · Distributional characterizations · Goodness-of-fit tests · Non-normalized statistical models · Probability distributions · Stein’s method

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