



Equivalence between adaptive Lasso and generalized ridge estimators in linear regression with orthogonal explanatory variables after optimizing regularization parameters

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Abstract

In this paper, we deal with a penalized least-squares (PLS) method for a linear regression model with orthogonal explanatory variables. The used penalties are an adaptive Lasso (AL)-type ℓ_1 penalty (AL penalty) and a generalized ridge (GR)-type ℓ_2 penalty (GR penalty). Since the estimators obtained by minimizing the PLS methods strongly depend on the regularization parameters, we optimize them by a model selection criterion (MSC) minimization method. The estimators based on the AL penalty and the GR penalty have different properties, and it is universally recognized that these are completely different estimators. However, in this paper, we show an interesting result that the two estimators are exactly equal when the explanatory variables are orthogonal after optimizing the regularization parameters by the MSC minimization method.

Keywords Adaptive Lasso · C_p criterion · GCV criterion · Generalized ridge regression · GIC · Linear regression · Model selection criterion · Optimization problem · Regularization parameters · Sparsity

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