



# Integral transform methods in goodness-of-fit testing, II: the Wishart distributions

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## Abstract

We initiate the study of goodness-of-fit testing for data consisting of positive definite matrices. Motivated by the appearance of positive definite matrices in numerous applications, including factor analysis, diffusion tensor imaging, volatility models for financial time series, wireless communication systems, and polarimetric radar imaging, we apply the method of Hankel transforms of matrix argument to develop goodness-of-fit tests for Wishart distributions with given shape parameter and unknown scale matrix. We obtain the limiting null distribution of the test statistic and a corresponding covariance operator, show that the eigenvalues of the operator satisfy an interlacing property, and apply our test to some financial data. We establish the consistency of the test against a large class of alternative distributions and derive the asymptotic distribution of the test statistic under a sequence of contiguous alternatives. We obtain the Bahadur and Pitman efficiency properties of the test statistic and establish a modified version of Wieand's condition.

**Keywords** Bahadur slope · Bessel function of matrix argument · Contiguous alternative · Diffusion tensor imaging · Factor analysis · Gaussian random field · Pitman efficiency · Zonal polynomial

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