



# Testing for normality in any dimension based on a partial differential equation involving the moment generating function

Norbert Henze<sup>1</sup> · Jaco Visagie<sup>2</sup>

Received: 14 January 2019 / Revised: 12 April 2019 / Published online: 16 May 2019  
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## Abstract

We use a system of first-order partial differential equations that characterize the moment generating function of the  $d$ -variate standard normal distribution to construct a class of affine invariant tests for normality in any dimension. We derive the limit null distribution of the resulting test statistics, and we prove consistency of the tests against general alternatives. In the case  $d > 1$ , a certain limit of these tests is connected with two measures of multivariate skewness. The new tests show strong power performance when compared to well-known competitors, especially against heavy-tailed distributions, and they are illustrated by means of a real data set.

**Keywords** Moment generating function · Test for multivariate normality · Direct sum of Hilbert spaces · Multivariate skewness · Weighted  $L^2$ -statistic

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The second author's work is based on research supported by the National Research Foundation, South Africa (Research chair: Non-parametric, Robust Statistical Inference and Statistical Process Control, Grant number 71199). Opinions expressed and conclusions arrived at are those of the authors and are not necessarily to be attributed to the NRF.

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✉ Norbert Henze  
henze@kit.edu

<sup>1</sup> Institute of Stochastics, Karlsruhe Institute of Technology (KIT), Englerstr. 2, 76313 Karlsruhe, Germany

<sup>2</sup> Department of Statistics, University of Pretoria (UP), Private Bag X20, Hatfield 0028, South Africa