Testing for normality in any dimension based on a partial differential equation involving the moment generating function

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Abstract

We use a system of first-order partial differential equations that characterize the moment generating function of the d-variate standard normal distribution to construct a class of affine invariant tests for normality in any dimension. We derive the limit null distribution of the resulting test statistics, and we prove consistency of the tests against general alternatives. In the case $d > 1$, a certain limit of these tests is connected with two measures of multivariate skewness. The new tests show strong power performance when compared to well-known competitors, especially against heavy-tailed distributions, and they are illustrated by means of a real data set.

Keywords Moment generating function · Test for multivariate normality · Direct sum of Hilbert spaces · Multivariate skewness · Weighted L^2-statistic

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