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# Detecting deviations from second-order stationarity in locally stationary functional time series

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## Abstract

A time-domain test for the assumption of second-order stationarity of a functional time series is proposed. The test is based on combining individual cumulative sum tests which are designed to be sensitive to changes in the mean, variance and autocovariance operators, respectively. The combination of their dependent $p$ values relies on a joint-dependent block multiplier bootstrap of the individual test statistics. Conditions under which the proposed combined testing procedure is asymptotically valid under stationarity are provided. A procedure is proposed to automatically choose the block length parameter needed for the construction of the bootstrap. The finite-sample behavior of the proposed test is investigated in Monte Carlo experiments, and an illustration on a real data set is provided.

**Keywords** Alpha mixing · CUSUM test · Autocovariance operator · Block multiplier bootstrap · Change points

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