



Nonparametric estimation of the cross ratio function

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Abstract

The cross ratio function (CRF) is a commonly used tool to describe local dependence between two correlated variables. Being a ratio of conditional hazards, the CRF can be rewritten in terms of (first and second derivatives of) the survival copula of these variables. Bernstein estimators for (the derivatives of) this survival copula are used to define a nonparametric estimator of the cross ratio, and asymptotic normality thereof is established. We consider simulations to study the finite sample performance of our estimator for copulas with different types of local dependency. A real dataset is used to investigate the dependence between food expenditure and net income. The estimated CRF reveals that families with a low net income relative to the mean net income will spend less money to buy food compared to families with larger net incomes. This dependence, however, disappears when the net income is large compared to the mean income.

Keywords Asymptotic distribution · Bernstein estimation · Copula · Cross ratio function · Hazard rate

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