



More good news on the HKM test for multivariate reflected symmetry about an unknown centre

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Abstract

We revisit the problem of testing for multivariate reflected symmetry about an unspecified point. Although this testing problem is invariant with respect to full-rank affine transformations, among the few hitherto proposed tests only a class of tests studied in Henze et al. (J Multivar Anal 87:275–297, 2003) that depends on a positive parameter a respects this property. We identify a measure of deviation Δ_a (say) from symmetry associated with the test statistic $T_{n,a}$ (say), and we obtain the limit normal distribution of $T_{n,a}$ as $n \rightarrow \infty$ under a fixed alternative to symmetry. Since a consistent estimator of the variance of this limit normal distribution is available, we obtain an asymptotic confidence interval for Δ_a . The test, when applied to a classical data set, strongly rejects the hypothesis of reflected symmetry, although other tests even do not object against the much stronger hypothesis of elliptical symmetry.

Keywords Test for reflected symmetry · Fixed alternatives · Affine invariance · Weighted L^2 -statistic · Elliptical symmetry

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